

THE EFFECTS OF  
THE CONTENT ENHANCEMENT MODEL  
IN COLLEGE ALGEBRA

BY

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In partial fulfillment of the requirements for the degree of  
Doctor of Philosophy.

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## **Abstract**

The purpose of this study was to investigate The Content Enhancement Model in the field of college algebra in a mid-western community college. The Content Enhancement Model is a teaching technique that teachers use to help students acquire the content information by helping them identify, organize, comprehend, and memorize material.

This study investigated the effects of The Content Enhancement Model on the college algebra achievement, mathematics confidence levels and students' perceived value of the Content Enhancement Routines on their college algebra achievement.

This study used a quasi-experimental design with control and experimental groups. Gender was also a factor.

The Content Enhancement Routines included Concept Diagram, Concept Comparison, and Concept Anchoring. The topics used in these routines included function, domain, slope, linear functions, and quadratic functions. The Content Enhancement Organizers included Course, Unit, and Lesson. The organizers were used for an introductory function unit, linear function unit and quadratic function unit in a college algebra course.

Statistically significant differences in mean algebra achievement post-test scores were found between students in the control and experimental groups in favor of the experimental group using the ANOVA. Significant differences were also found in algebra achievement posttest scores on a sub-topic, slope in favor of the experimental group.

This study supports the use of the Content Enhancement Model as an effective way to increase overall algebra achievement in a community college setting.

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Special thanks go to The Center of Research and Teaching for their years of research to develop the Content Enhancement Routines and especially Dr. Janis Bulgren. She helped me in so many ways. She took a personal interest in combining The Content Enhancement Series with the study of algebra. Our many conversations not only inspired me to move forward in using The Content Enhancement Series, but also allowed me to look at the algebra curriculum in innovative ways.

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## **Chapter I**

### **Introduction**

#### **Background of the Study**

For many educators, studying mathematics, as a discipline, requires mainly a mastery of symbolic manipulation of expressions, equations, and inequalities. The basic instruction involves rote memorization of procedures, using homework to master each topic. However, it is the belief of this researcher that conceptual knowledge is underutilized in most mathematics classrooms. In order to accommodate the changing demands of the workforce The American Mathematical Association of Two-Year Colleges published two documents: 1) Crossroads in mathematics: Standards for introductory college mathematics before calculus (American Mathematical Association of Two-Year Colleges [AMATYC], 1995) and 2) Beyond crossroads: Implementing mathematics standards in the first two years of college (AMATYC, 2006) to channel mathematics education in a different direction for the changing workforce. Other organizations recommend teaching conceptually (The National Council of Teachers of Mathematics [NCTM], 1989; National Research Council [NRC], 1989). The goal of this study is to promote the understanding of conceptual mathematics through The Content Enhancement Model. This will help to change the teaching of mathematics to meet the demands of our current and future society. An additional goal of this study is to support the AMATYC documents (AMATYC 1995, 2006). These documents provide a set of standards to improve the teaching of mathematics education and to encourage more students to study mathematics.

## Statement of the Problem

One mission of a community college is to prepare students for transfer to a four-year institution. Students come from several types of diverse backgrounds; many are older, under-prepared for academic work, and have learning disabilities (AMATYC, 1995). Therefore, it is necessary to find quality teaching techniques to help instruct all students attending post-secondary institutions. This is particularly true for mathematics (National Research Council [NRC], 1990). Other reports have made recommendations for needed changes in the classroom. The authors of *Crossroads in Mathematics* presented new standards designed for two-year colleges. These standards for pedagogy accommodate the use of teaching strategies that provide for more interaction between student and instructor. In this way students can better construct their own knowledge (AMATYC, 1995). The Content Enhancement Model is one way to teach academically diverse groups of students. This model helps all students learn by addressing the students' difficulties with organizing, understanding, and remembering information. All of these skills are necessary for students studying mathematics.

The three components of The Content Enhancement Model are devices, routines, and procedures and are associated with instruction (Bulgren & Lenz, 1996). Content Enhancement helps instructors plan their programs and choose their content in order to use the instructional devices to help students learn. With The Content Enhancement Routines instructors design the main ideas and relationships of the content to be more concrete for students. Instruction is interactive and carried out in a partnership with the students. This keeps the students involved in the learning process (Bulgren & Lenz, 1996).

The plan to teach academically diverse students using The Content Enhancement Routines requires the instructors to contemplate about how they select and organize the material.

In this manner they meet not only the needs of individual students, but also of the entire class. The instructors are more active in identifying what is really important for the student to learn. They create a curriculum to meet the needs of all students by strengthening the content of what students need to learn (Bulgren & Lenz, 1996).

### **Purpose of the Study**

The purpose of this study is to determine the effects of The Content Enhancement Model on teaching college algebra to students at a community college. The Content Enhancement Model was developed at the Center for Research on Learning, University of Kansas. This study examines the effects of The Content Enhancement Model in three ways. First, this study examines the effects of this model on algebra achievement posttest scores and mathematics confidence posttest scores of community college students. Second, this study determines the correlation between scores on the mathematics confidence scale and the algebra achievement posttest scores. Third, this study determines the correlation between the students' perceived value of The Content Enhancement Series and the algebra achievement posttest scores. The duration of this study was about five weeks after a review of basic procedural skills.

### **Significance of the Study**

This study is important because many teachers from the elementary school to the university level use the lecture method to teach mathematics. This usually involves using the method of rote learning to teach basic skills. This involves emphasizing rules and procedures rather than using a method that teaches conceptually for understanding. Content Enhancement Routines are a series of steps to help instructors use research-based materials to address the new pedagogical standards from The National Council of Teachers of Mathematics and The American Mathematical Association of Two-Year Colleges. The main pedagogical standards



supported by the mathematical organizations that The Content Enhancement Series supports are: 1) interactive and cooperative learning, 2) connecting with other experiences, 3) using multiple approaches for instruction and 4) the role of lecturing (AMATYC, 1995; NCTM, 1989). The Content Enhancement Series has many routines to support the main aspects of the content subject curriculum. All of the routines are carried out in a partnership between the student and the instructor to make the lecture method more effective for teaching. During this interactive partnership, the instructor is able to determine the students' prior knowledge and experiences to help them form new connections, which leads to the discovery of new topics (Bulgren & Lenz, 1996).

## **Research Questions and Hypotheses**

### **Research Questions.**

1. Is there a correlation between the means on the mathematics confidence pretest scores and the algebra achievement posttest scores?
2. Is there a difference in the means on algebra achievement posttest scores for students in the experimental and control groups assuming no prior differences? Is there a difference in the means on the mathematics confidence posttest scores for students in the experimental and control groups assuming no prior differences? The means for the groups are averaged across male and female students.
3. Is there a difference in the means on algebra achievement posttest scores on the linear and quadratic questions between the control and experimental groups?
4. Is there a difference in the means on algebra achievement posttest scores on the domain questions between the control and experimental groups?

5. Is there a difference in the means on algebra achievement posttest scores on the slope questions between the control and experimental groups?
6. Is there a correlation between the means on the students' perceived value scores of The Content Enhancement Series and the algebra achievement posttest scores?

### **Hypotheses.**

For hypothesis one, there will be a significant positive correlation between the means on the mathematics confidence pretest scores and the algebra achievement posttest scores. For hypothesis two, the experimental group will have higher means on algebra achievement posttest scores than the control group. The experimental group will have higher means on mathematics confidence posttest scores than the control group.

For hypothesis three, the experimental group will have higher means on algebra achievement posttest scores on linear and quadratic questions than the control group. For hypothesis four, the experimental group will have higher means on algebra achievement posttest scores on domain questions than the control group. For hypothesis five, the experimental group will have higher means on algebra achievement posttest scores on the slope questions than the control group.

For hypothesis six, there will be a significant positive correlation between the means on the students' perceived value scores of The Content Enhancement Series and the algebra achievement posttest scores.

### **Organization of the Study**

This research study includes five chapters. Chapter 1 describes the background of the study, statement of the problem, purpose of the study, significance of the study, research questions and hypotheses, and organization of the study.

Chapter 2 includes a review of the literature consisting of introduction, needs in mathematics education, gender, confidence, cognitive learning and instruction, content enhancement, mathematics, and overview of content. Chapter 3 describes the methodology used for this study including introduction, design of the experiment, selection of subjects, instrumentation, data collection, data analysis, and summary.

Chapter 4 is the presentation and analysis of data. It includes introduction, descriptive statistics and the results of testing the research questions. Chapter 5 is the summary, discussion, and conclusions. It includes introduction, summary of the analysis, discussion of the findings, limitations of the study, implications for practice, recommendations for further research and conclusion.

## **Chapter II**

### **Review of the Literature**

#### **Introduction**

The opportunity to know and study algebra is a right of every student (NCTM, 1989). Therefore, every student deserves a classroom experience to learn mathematics for understanding (National Council of Teachers of Mathematics [NCTM], 1990). This is important because the knowledge of algebra may expand the student to be more curious and creative. This leads ultimately to the ability to compete in our technological world (Heid, 1996; Hovis, Kimball, & Peterson, 2003; NCTM, 1990). The study of algebra is not just a prerequisite for the study of advanced mathematics. It is a collection of concepts and rules needed for the symbolic system, and this system is needed to show relationships. The study of these relationships is necessary to make inferences among variables and to make predictions concerning these relationships (Usiskin, 1988)

#### **Needs in Mathematics Education**

##### **Background.**

Students who enter post-secondary educational institutions have different backgrounds in terms of their mathematics preparation (AMATYC, 1995). Some students enter with a solid background of mathematics from high school and are ready for the calculus series when they enter their first year of post-secondary education. Others need to further their mathematics background by beginning at the level of college algebra or lower in order to prepare for calculus; still others need to study mathematics below calculus to satisfy prerequisites for entering certain career fields (AMATYC, 1995). The American Association of Two-Year Colleges' Standards described in *Crossroads in Mathematics* are designed for students who have not had adequate

preparation for entering the calculus series. A majority number of community college students in mathematics are not ready for calculus and less than 20% of all community college students study pre-calculus (Albers, Loftsgarden, Rung, & Watkins, 1992).

### **Growth in Mathematically Based Occupations.**

Students at two-year colleges are preparing to enter job markets that increasingly call for a solid background of mathematics courses (AMATYC, 1995; NRC, 1989). More new jobs will require higher levels of mathematics knowledge in the future and this knowledge is necessary for the expansion of the mathematics and science occupations (NCR, 1990).

### **Algebra in a Technological World.**

Technology is changing the way we look at algebra. The study of algebra will no longer consist of mainly perfecting the techniques of symbolism (Heid, 1996). This changing view now emphasizes conceptual understanding along with procedural ability and mathematical modeling (Heid, 1996). Using technology to do mathematical modeling is one way students will value studying algebra (Heid, 1996). Technologies that use a symbolic mathematics system and arithmetic are important for students to experience during their study of mathematics (Heid, 1996; Hovis et al., 2003).

Two main concepts in using technology for modeling are function and variable. These concepts are vital for studying relationships in real-world applications (Heid, 1996; Herriott, 2006).

### **Diversity.**

Minorities and women will constitute a larger share of the new workers in the future, because of the significant recent changes in ethnic and racial composition of students (NCR, 1989). As more people from these under-represented categories prepare to enter the work force,

finding educational methods that allow all students, with varying levels of mathematics preparation, to achieve in mathematics becomes important. Mathematics should not be the reason that students are forced to leave schools or are prevented from entering careers (NCR, 1989). Diversity includes also the levels of academic learning (Bulgren & Lenz, 1996).

The reform movement stresses that the achievement of these minority groups must be addressed. These students of color, second learners, females, and low-economic groups must achieve the necessary mathematical skills to succeed in our world today (Martin, 2004).

### **The Community College.**

The establishment of community colleges has been a major institution in higher education in the twentieth century (League for Innovation in the Community College, 1981). Community colleges offer transfer programs, general education, technical and vocational education, and developmental education programs. Since community colleges offer open-door access and flexibility for students, many high school graduates continue their education by entering two-year college institutions particularly when both part-time and full-time students are counted (AMATYC, 2006; League for Innovation in the Community College, 1981). Many of these students enter with low academic skills. This increase in under-prepared students has presented a challenge for academic faculty at community colleges. Instructors at two-year institutions are seeing large ranges of diverse skill levels among students in a single classroom. This trend requires a need to shift from traditional teaching methods to methods that are more appropriate for students with different skill levels (AMATYC, 1995, 2006).

### **Instruction in the Classroom.**

Traditionally, teachers have been perceived as presenting information by lecture, and the students' role was to absorb the information. This is a teacher-centered style where knowledge is

transferred from instructor to student (AMATYC, 2006). Students show their “learning” by regurgitating the content on exams. However, this method does not effectively allow students to change the information to meaningful knowledge. In order to make this kind of change the students must link the new information to their prior knowledge. Ausubel’s theory of meaningful learning assumes that thinking is based on concepts (Ausubel, 1968). For this learning to be meaningful, the student must link this learning to their prior knowledge (Novak & Gowin, 1984).

In mathematics classrooms the traditional model of teaching involves a method where instructors state the mathematical concept and follow up with individual seatwork. This involves low level drill-and-practice exercises. A large number of mathematics instructors continue to follow this method (Olson, 1999). This type of mathematics instruction does little to promote the understanding of conceptual mathematics into meaningful knowledge. The drill-and-practice method does not lead to transferring the learning into other areas (Resnick & Ford, 1981). This traditional method of teaching mathematics is not serving the needs of diverse students today because it fails to address the issues of different skill levels and different learning needs (AMATYC, 1995; NCTM, 2000). It is considered by students too abstract and of little interest to them (Darken, 1995). Two of the recommendations made by *Crossroads in Mathematics* for alternative methods in teaching mathematics are: 1) that instructors emphasize the meaningful relationships among mathematical concepts and 2) that students participate actively in order to acquire effective instruction (AMATYC, 1995, 2006). An instructor at a community college offered the suggestions for teaching developmental mathematics. The suggestions were 1) identify and develop the key mathematical concepts, 2) design tasks to engage students, 3) ask students questions that promote mathematical thinking, and 4) help students develop mathematical power, not just finding correct answers (Stump, 2009).

## Gender

Before 1979 there were some differences in mathematical performance in favor of the male students (Schonberger, 1980). However, there are greater differences in the rates of participation in upper level mathematics courses between male and female students (Schonberger, 1980). Students perceived that teachers were more supportive to males than females (Schonberger, 1980). Also, one variable measured by Fennema and Sherman found that both male and some female students thought mathematics was a male domain (Fennema & Sherman, 1978). The gender difference in participation in mathematics classes continued to be observed in upper level mathematics courses, and academic performance in these courses tended to favor males (Leder, 1992). A possible explanation for this difference level includes several variables including schools, teachers, peer groups and society (Leder, 1992). Some cognitive variables include intelligence and spatial ability. Some internal variables include confidence, fear, motivation and persistence. In several studies, these gender differences often show significance in students' performance on mathematical tasks. For these reasons, it is important to continue to examine gender's role in studies (Leder, 1992).

Differences in mathematical achievement for gender are a cause for concern. This is especially true in college and career choices related to the mathematical sciences (Fox, 2001). If females are under-represented in upper level mathematics classes, they are limiting their career options and limiting their future long-term quality of life. If mathematics is taught in a different way, females may be encouraged to understand and question the mathematics that drives how political decisions are made. This may also support their confidence to choose careers requiring mathematics (Tyrrell et al., 1994). This notion is emphasized in the NCTM *Curriculum and Evaluation Standards*. This document states that mathematics has become a critical filter for



career choices. This, in turn, controls who is able to participate in our society. In society, today, gender equity has become an economic necessity (Albers et al., 1992; AMATYC, 1995; NCTM, 1989). Educators are challenged to change this situation and to promote academic excellence and equity for all students. This will help to meet the needs of the new century (Croom, 1997).

Anderson (2007) argues that to realize the vision in mathematics education with respect to learning and equity, research must be studied to include not only the classroom issues, but also broader policy levels. If this is not done, the vision of school mathematics will not happen (Anderson, 2007).

### **Confidence**

The importance of self-confidence in doing mathematics was addressed by Keppel (1963). Average students could do mathematics if they remembered a few of the basic concepts (Keppel, 1963). For students to remember these basic concepts, it was an important goal of mathematics education at that time to build the students' self-confidence in their mathematical abilities. In conclusion, the study of mathematical basic ideas or concepts helped students to have the confidence to rely on their mathematical abilities (Keppel, 1963). Gaining confidence is a general goal for students studying mathematics (NCTM, 1989). Confidence is a belief about one's attitude towards doing mathematics (Reyes, 1980). A major objective of curriculum in mathematics is building self-confidence that is based on success (NRC, 1989).

At the college level, the instructor's role for improving students' self-confidence in their ability to learn mathematics is meaningful (NCTM, 2000). If students' confidence is high, then positive attitudes toward mathematical tasks will also be high (Linn & Hyde, 1989). As a result of this positive attitude, higher achievement in mathematics is anticipated. In previous studies, for many students, confidence was associated with enrollment in elective mathematics courses

(Linn & Hyde, 1989). At the university level, women were not as confident as men in their ability to obtain a PhD. (Mura, 1987).

There are three types of common student goals that students can exhibit. Each type can potentially have an effect on how the student approaches the subject (Dweck, 1986, Middleton & Spanias, 2002). The first type is a learning goal and it is associated with the mastery of a task. The second type is a performance goal and it is associated with gaining a favorable outcome. The third goal is called work avoidance (Middleton & Spanias, 2002). The degree of student confidence is directly related to the behavior of the student in approaching his or her goals (Dweck, 1986). A high level of confidence is associated with a mastery-oriented behavior. If confidence is high, and the focus is on learning as a goal, mastery learning is the result. If confidence is low, and the focus is on performance, it results in feelings of helplessness, and the student tends to avoid challenges (Dweck, 1986). This approach is consistent with the standards set by both the National Council of Teachers of Mathematics and the American Mathematical Association of Two-Year Colleges. The standards of these two organizations emphasize a goal of learning over a goal of performance (AMATYC, 1995; NCTM, 2000). The learning of algebra is a goal that two-year institutions should have for all their students (AMATYC, 1995). Confidence is an important aspect of one's learning environment. Confidence in learning mathematics stems from an individual's belief that one is able to learn new topics in mathematics. This confidence can be instilled in students by emphasizing learning over performance. Confident students also believe they can achieve their performance goals by doing well in their mathematics classes and doing well on tests. This level of confidence has been shown to be strongly associated with the ability to learn mathematics (Reyes, 1980).

## **Cognitive Learning and Instruction**

Learning theories contribute to our understanding of the many issues that affect the education of students. These theories have guided how instructors respond to the many challenges that they encounter on a daily basis with their students. By knowing the theoretical background, instructors can approach student learning situations with a frame of reference. In this way instructors can make better decisions throughout their teaching careers (Hohn, 1995).

Ausubel contributed many ideas to the advance learning theories. He defined meaningful learning as a process in which not only the material is meaningful, but also the material has meaning for the individual student. The learned material relates to the student's existing concepts and this forms the basis for significant new relationships. He contrasted this meaningful learning to rote learning, which is verbatim memorization with no outcome of learning (Ausubel, 1963). Ausubel argued that meaningful learning is based on the belief that humans use concepts in thinking (Ausubel, 1968). A student's prior learning is one of the main factors in what a student can learn (Ausubel & Robinson, 1969). What a student knows influences what he learns in these two ways: 1) prior knowledge indicates the readiness of the subject, and 2) the experience with the subject also contributes to the readiness of learning new material (Ausubel & Robinson, 1969). New knowledge, then, is constructed by observing events or objects through the concepts that a student already has acquired. These events or objects that have meaning for the student are linked to the prior concepts the student already has (Novak & Gowin, 1984).

Early learning theorists made contributions to the development of instructional techniques that are used by the constructivist. John Dewey, an educational theorist, proposed that schools place students in meaningful activities where group work is required to solve

problems (Dewey, 1958). He argued that merely relating new ideas to students was ineffective for problem solving. He thought the emphasis should be on a method that required reflective thinking or inquiry. Jean Piaget (1970) argued that a child constructs a schema that develops into concepts. These concepts are interrelated and organized to form a network or a cognitive structure (Piaget, 1970). Vygotsky (1978) emphasized the fact that learning primarily takes place through interactions with others (Vygotsky, 1978). He recognized a key factor in social learning was a young student's ability to learn by imitation. He also stated that if a student receives help with a task, the student may be able to complete the task independently at a later time. Vygotsky called this help the concept of zone of proximal development (Vygotsky, 1978).

Some theories focused on the structure of knowledge, and that learning how things are related is learning the structure of a subject. Ausubel promoted this theory that concepts are learned in a hierarchical structure (Ausubel, 1963). A new concept, he argued, is learned by linking it to an already known concept. The direction of the linkage depends on the learner's ability to create an association between specific concepts and higher-order concepts. The student's process of learning in this way is helped by a teaching technique in which the student is presented with advance organizers or anchoring ideas. These techniques are usually introduced to the students before learning the material so they can establish meaningful learning (Ausubel & Robinson, 1969). The way knowledge is structured also influences the learning. Loosely connected information is not as effective as well structured information when learning new material (Hiebert & Carpenter, 1992).

Janis Bulgren (1987) a researcher at KU-CRL, has examined the way in which concepts are defined and analyzed for use in secondary classrooms. She researched the definition of a concept and found that a concept includes categories with relationships. Categories include the

grouping of events, ideas and objects. Relationships are the connections derived through categorization. Bulgren identified the ways in which a concept can be represented. The name of a concept is a word or group of words used for identification. This helps to create additional vocabulary for students. The characteristics of a concept are the features, traits or qualities used for identifying the concept. Determining a concept involves the process of sorting the attributes of the concept. Examples of a concept are sometimes used when concepts are taught. Likewise non-examples are used when one or more attributes of the concept are missing. A definition of a concept includes: 1) naming a higher class of concepts that includes the one being studied, 2) a listing of the characteristics, 3) a listing of the examples and non-examples, and 4) specifying the relationships among the characteristics (Bulgren, 1987).

### **Constructivism.**

Constructivism has been divided into several types (Watson & Mason, 2005). One of these is “simple constructivism.” In “simple constructivism” the learners develop a meaning of the material. Another type of constructivism is “radical constructivism” in which the learners are trying things out to put meaning to their past and future experience. In “social constructivism” learners are interacting to create meaning. In a fourth meaning, “mathematical constructivism,” learners construct a meaning with specified constraints. When the learner does this, self-confidence is gained and the learner is able to re-construct meaning to yield new construction (Watson & Mason, 2005).

When constructivism is applied to teaching, it is no longer assumed that meaningful understanding will occur if knowledge is just passed on to the student. Teaching is more than merely telling. The instructor must restructure and reorganize the material along with the view points of the student. Then, the instructor must provide experiences to assist the student in re-

constructing their own meaning of the new concepts. This is done by a reflective, thoughtful process which includes some human action by the student such as counting, folding, ordering, comparing, etc. (Confrey, 1990).

Content enhancement is not only compatible with constructivism, it also recognizes the instructor as organizing and planning the instruction (Bulgren & Lenz, 1996). Both of these ideas allows content enhancement to be under the paradigm of functionalism (Lenz & Mercer, 1992). This is regarded as functional constructivism (Bulgren & Lenz, 1996).

### **Teaching Techniques.**

Bulgren (1987) identified teaching techniques designed to help in the instruction of concepts. These techniques are responsive to the needs of students by helping the student to learn content in a way that promotes conceptual and higher-order thinking. These techniques are responsive to the needs of the instructor, by using techniques that can be incorporated into the traditional lecture format. These techniques include using advance organizers, graphic organizers, and planning for interaction between the instructor and the students (Bulgren, 1987).

An advance organizer consists of material that is presented to the student before the specific learning task. An advance organizer specifies a broad and abstract overview of the learning task (Ausubel & Robinson, 1969). This advance organizer strengthens both the vertical connections of the structured concepts as well as the horizontal connections. This provides a scaffolding method that allows students to structure their new information (Ausubel, 1963). Adolescents with learning disabilities in regular secondary classrooms improved their retention of content knowledge using advance organizers. However, the improvement occurred only if instructors taught the students to use the advance organizers in an effective manner. A set of twelve behaviors were identified to help students effectively use the advance organizers. These

behaviors included not only background information of the material but also motivational behaviors for students. The behaviors used by the instructor to prepare the students for using advance organizers are either written or verbal instructions (Lenz, Alley, & Schumaker, 1987).

Graphic organizers are visual ways to present material to students in order to help them identify, organize and comprehend concepts, and then show how these concepts are connected and related. The relationships are constructed between two or more items of the material or content. Graphic organizers may be used in a variety of formats. They are hierarchical, comparative, directional, and representative (Hudson, Lignugaris-Kraft, & Miller, 1993). The hierarchical format focuses on a main topic, and all the other information flows out from this center topic. This format is useful for teaching new concepts and for reviewing previously acquired concepts. This format is also good for guiding students in practice activities. The comparative format illustrates the associations between two or more concepts, which are compared and contrasted. The directional format presents material in sequential steps showing relationships and can be used by the teacher to introduce new concepts. It can also be used to review and access previously learned concepts. Finally, the representational format includes specific models such as diagrams or pictures that represent the new concept. It also specifies the relationships. A five-step method to explain the process and procedure for students using graphic organizers includes explanation, modeling, application, reflection, and independent practice (Cassidy, 1989). This method shares some of the same components that were previously identified (Lenz, 1997).

There is evidence that visual graphics increase not only comprehension but also higher-order thinking skills for low-achieving students with weak thinking strategies (Mayer, 1989). The effect of graphic organizers has been studied with middle and high school students in

science and social studies classes. The subjects consisted of general, remedial and learning disabled students. Instructors interacted with the students in using graphic organizers, and students then filled in appropriate blanks on the diagram. This use of the graphic organizer was shown to be positive for all types of students. Adapting the content information to the graphic organizers increased the possibility of the learning disabled and lower-achieving students to acquire mastery of the content information (Lovitt & Horton, 1994).

Bulgren designed, studied, and evaluated the effectiveness of the Concept Diagram and the Concept Teaching Routine in regular secondary classrooms that included learning disabled students. All students scored significantly higher under the Concept Teaching Routine. The students also showed an increase in the amount of notes recorded during the interactive teaching phase. Aside from significant concept attainment, this study also integrated the Concept Teaching Routine into regular classrooms (Bulgren, Schumaker, & Deshler, 1988; Kissam & Lenz, 1994).

Planned Interaction between teachers and students is an important technique for teaching concepts. One representative technique was described and defined through a strategy called the Reciprocal Teaching Technique, in which the students and teachers ask questions to each other about the material and modeled the search to find solutions (Palincsar & Brown, 1984). This strategy then develops into questions about the meaning of text. This part of the process included student-generated questions, summaries of the content, clarification of points, and prediction of future subject matter (Palincsar, Ransom, & Derber, 1989). Learning disabled students are good candidates for this form of instructor-student interaction, because learning disabled students tend to have passive learning styles. To promote active learning, instructors asked students about background knowledge, focused students on important content and asked



students questions (Wong, 1985). This technique is adaptable to the traditional lecture format in secondary content classrooms. Student-instructor interactions and explorations are built into the instruction. This is based on the Concept Mastery Routine. The Concept Mastery Routine includes an interaction with students while addressing the overall concept, characteristics, examples, and determining a definition (Bulgren, Schumaker, & Deshler, 1988)

## **Content Enhancement**

### **Introduction.**

The student population of two-year community colleges is changing to include more levels of academic performance and also a greater diversity in culture, gender, beliefs, goals, and learning needs (AMATYC, 1995). A definition for this significant diversity of ability levels is described by the acronym HALO (high, average, low, and others) (Kissam & Lenz, 1994). This increasing spread of ability levels in academic diversity challenges instructors to find new methods to teach not only each individual in the classroom, but also to cover the content of the subject for all the students (Bulgren & Lenz 1996; Lenz, Marrs, Schumaker, & Deshler, 1993). Typically, most instructors teach to the “B” students. This results in a disengaged class because not all students feel accepted and connected to the content material (Bulgren, Schumaker, & Deshler, 1993).

One of the ways of providing for all students in this diverse academic population to gain mastery of the material is to use the instructional approach, The Content Enhancement Routines (Bulgren & Lenz, 1996; Lenz et al. 1993). The Concept Mastery Routine is one of The Content Enhancement Routines. This approach is a set of teaching techniques that meets both class needs and individual needs. It keeps the integrity of the content by selecting and changing critical

information to promote learning. This technique carries out the instruction with the students and the instructor (Lenz & Bulgren, 1995).

To compensate for the diverse abilities of students, instructors can use Content Enhancement Routines to help students learn the important subject material. Content Enhancement Routines help students identify, organize, comprehend, and memorize the critical content information of a subject (Lenz, Bulgren, & Hudson, 1990). This approach is a way of teaching that meets individual and class needs while keeping the wholeness of the content. Content enhancement also selects critical features of the content and changes this information in a manner that promotes learning. It also carries out the instruction in an interactive way between the instructor and the students (Bulgren & Lenz, 1996; Lenz & Bulgren, 1995).

### **Principles.**

Content Enhancement is founded on four fundamental principles. These principles are instructional leadership, expert instruction, functional constructivism and learning partnership. These principles are research-based ideas that include curriculum, instruction, educational psychology, and special education. They serve as guides for instructors using The Content Enhancement method (Bulgren & Lenz, 1996).

The first of these principles is *instructional leadership*. To assist students in processing information, it is necessary for instructors to be able to organize the content and to interact with students. The ability to organize and to interact with students places the instructor at the center of the decision making process. The elements that are continually interacting in this process are: 1) the learners' characteristics, 2) the learning activities, 3) the nature of the task, 4) the nature of the materials, and 5) the instructional agent (Tunure, 1986). The instructor's ability to

coordinate these different elements is an important part of the instructional process in order for students to be successful in learning material.

The second of these principles is *expert instruction*. The instructor is responsible for not only knowledge of the content and the content relationships, but also the presentation of the content in an understandable way for the students. To achieve this goal the instructor must be aware of: 1) various types of information and how information can best be presented to the student, 2) the problems that students have in learning the content, and 3) various ways of making the content information meaningful to the student.

The third of these principles is *functional constructivism*. This principle assumes that the approach to education emphasizes the student's internal constructive processes to master the content (Gagne, 1985). This view sees students and instructors as partners in constructing meaning from the content information. Since the instructor is responsible for monitoring the learning process, the teaching must remain flexible to meet the needs of the student.

The fourth of these principles is *learning partnership*. This principle assumes that the student and the instructor are partners in the learning process (Bulgren et al., 1993). Since this relationship is a partnership, the instructors must recognize the student's learning strategy needs (Bulgren & Lenz, 1996). This is particularly important because the learning disabled student is not always aware of the learning strategies (Lenz et al., 1987). The instructor can use content enhancement to guide the student in the learning process.

### **Components.**

The Content Enhancement process is composed of three components that are designed to meet the needs of student learning. These three components are devices, routines, and procedures in strategic teaching. This model of teaching helps instructors meet the needs of all

students. This is done by selecting appropriate techniques to develop and organize instruction that covers the content. This enhances learning for students of diverse academic backgrounds (Bulgren & Lenz, 1996).

The first of these components is *devices*. A teaching device is an instructional tool to promote learning. It is a tool to help the learner with organizing, understanding, recalling, and applying information. In addition, a teaching device can help teachers: 1) focus on specific points, 2) make learning explicit, 3) prompt elaboration on a point, and 4) make ideas and relationships concrete (Bulgren & Lenz, 1996).

Teaching devices can be either verbal or visual. Verbal examples of organization are summaries, chunking and advance organizers while visual examples are outlines, graphic organizers, tables, grids and flowcharts (Bulgren & Lenz, 1996; Lenz & Bulgren, 1995). To promote understanding, analogies, synonyms, antonyms, examples and comparisons are examples of verbal devices while symbols, concrete objects, pictures and diagrams are visual devices. Verbal stories and scenarios and visual films and videos can be used for descriptions or telling stories. Verbal teaching devices include role playing or dramatic readings while motion and movement include visual demonstrations. Verbal devices for recall can be acronyms or keywords, while visual devices can be sketches. Visual devices can easily be incorporated into the lecture format, with presentations such as the graphic organizers, diagrams, tables, outlines, films and demonstrations. Verbal devices help to draw students to focus on important points, and can also be included in the traditional lecture format. Some of these visual devices are research based such as concept webbing and a concept analysis table (Bulgren & Lenz, 1996; Lenz & Bulgren, 1995). The Concept Diagram, the center-piece of The Concept Mastery Routine is an example of a visual device (Bulgren et al., 1993). These devices show students the

organization of the material in a visual way. Since some students are not aware that the teachers are using devices, it is often necessary for teachers to make students aware of the importance of these devices (Lenz et al., 1987).

The second component of Content Enhancement is the use of *routines*. A teaching routine is a set of procedures the instructor uses to integrate the content information. This is a set of integrated instructional procedures revolving around a teaching device that promotes learning of critical content area knowledge. The purpose of the routine is to help the learner acquire the knowledge by structuring the way he learns. The instructor can plan and use the routines to anticipate and address student difficulties (Bulgren & Lenz, 1996). These Content Enhancement Routines fall into three areas: 1) setting the stage, 2) learning conceptual knowledge, and 3) mastering factual knowledge. They focus on organization, concepts, recall, and applications (Bulgren & Lenz, 1996; Lenz, 1997).

The Content Enhancement Routines are in two categories. One category includes the teaching routines. The three teaching routines are Concept Anchoring Routine (see Appendix A: Concept Routines), Concept Mastery Routine, and Concept Comparison Routine (Bulgren & Lenz, 1996; Bulgren et al., 1995; Bulgren et al., 1993, 1994; Tollefson, 2005). The second category includes organizer routines. The three organizer routines in The Content Enhancement Series are Course Organizer Routine (see Appendix B: Course Organizer Routines), Unit Organizer Routine (see Appendix C: Unit and Lesson Routines), and Lesson Organizer Routine (Bulgren & Lenz, 1996; Lenz, Bulgren, Schumaker, Deshler, & Boudah, 1994; Lenz et al., 1993; Lenz, Schumaker, Deshler, & Bulgren, 1998; Tollefson, 2005). All of these routines incorporate the use of graphic devices embedded in specially designed instructional routines.

The organizer routines make students notice how information is structured. It also makes the student aware of any prior information that will help in the presenting of the content. The organizer routines are most effective if they are made explicit to the student before, during, and after the instruction. The more the students are aware of the use of the organizers, the greater the benefit of using the routines for enhanced learning for the student (Bulgren et al., 1988).

***The Course Organizer Routine.*** (Lenz & Deshler, 2004; Lenz et al., 1998) is designed to begin the course when the course or semester starts. It gives the students an overview of what the course covers with a graphic device. The Course Organizer reminds students during the semester where they are in terms of the entire course. The main aspects of the Course Organizer are: 1) the course that names and paraphrases what the course is about, 2) the course questions that cover the broad concepts, 3) the course standards that name the grading standards and processes, 4) the critical concepts that will be covered, 5) the content map that graphically shows the content, 6) the community principles that give the parameters for the class community, 7) the learning rituals that name the different routines and strategies, and 8) the performance options that allow for the diverse needs of the students. The Course Organizer is designed to be used consistently by the instructor and students to show the progress of the course (Lenz & Deshler, 2004; Lenz et al., 1998).

***The Unit Organizer Routine.*** (Lenz et al., 1994; Lenz & Deshler, 2004) is designed to graphically organize the subject area content of a unit. This visual device, Unit Organizer, has of the following sections: 1) the current unit that names the new unit, 2) the last and next units that position the content with other units, 3) the bigger picture that names the present concept, 4) the unit map that graphically shows the meaning of the unit, 5) the unit relationships that can be compared and contrasted, 6) the unit questions that students should answer, 7) the unit schedule

that names the assignments for students, and 8) an expanded map and more questions about the content. An important aspect of this routine is the interaction between the instructor and the students. They work together in order for the students to discover the relationships between the concepts (Lenz et al., 1994).

***The Lesson Organizer Routine.*** (Lenz & Deshler, 2004; Lenz et al., 1993) is designed to promote interaction between the instructor and students in order to make the learning process better. Its main aspect is the visual device, the Lesson Organizer. It has several sections that provide information to the students for certain purposes. The sections are: 1) the lesson topic that names the main idea, 2) the relationships that are represented in the material, 3) the strategies needed for the lesson, 4) a graphic display of the parent unit, 5) the graphic display of the lesson, 6) the challenge question to attract the interest of the student, 7) the self-test questions to allow the students to review the lesson, and 8) the tasks over the lesson that students need to complete. In addition to the visual device, there are directions to help the instructor complete the sections when preparing the device. Instructors also choose and organize the content, plan the presentation, and decide the applicable tasks and questions for the students (Lenz et al., 1993).

When the students are aware of the organization and structure of the subject area material and when they are aware of the necessary prior knowledge, then the students are ready to master the concepts of the subject area content. Understanding concepts and relationships involves higher-order thinking (Bloom, 1984). To help this phase of the student's learning process the instructor needs to give students a method for helping to master higher-order thinking (Lenz & Deshler, 2004). This is done by integrating and storing subject area information. To achieve higher-order thinking the students must follow these steps: 1) receive the content, 2) recognize and organize relationships in the material, 3) retrieve relevant prior knowledge, 4) choose what

information to remember, and 5) move this information into their content networks to make conclusions. To help the student achieve this processing of information the instructor must use a conceptual method of teaching (Lenz & Deshler, 2004; Lenz et al., 1998). Some of The Content Enhancement Routines have been used in different ways to enhance the understanding of subject area material. These routines include the Concept Mastery Routine, The Concept Anchoring Routine, and The Comparison Table Routine (Bulgren & Lenz, 1996).

***The Concept Anchoring Routine.*** (Bulgren, Schumaker, Deshler, 1994; Lenz & Deshler, 2004) is a visual device to introduce and anchor a new concept to a prior concept which is familiar to the students. The Concept Anchoring Table ends with several sentences expressing the student's understanding of the new concept. The sections include: 1) naming the new concept, 2) naming the known concept, 3) the key words representing the known concept, 4) the characteristics of the known concept, 5) characteristics of the new concept, 6) common characteristics of the known and new concepts, and 7) sentences that show an understanding of the new concept. The instructor chooses a known concept which is one all the students have previous knowledge (Bulgren et al., 1994).

The Concept Anchoring Routine increased student performance for high-achieving, normally achieving and at-risk students in a diverse secondary content classroom where students were developing science skills (Bulgren, Deshler, Schumaker, & Lenz, 2000).

***The Concept Mastery Routine.*** (Bulgren et al., 1993; Lenz & Deshler, 2004) is designed to graphically represent the main parts necessary in analyzing a concept. It has several sections that are intended to lead to the concept definition. The sections are: 1) the name of the concept, 2) the name of the broader or supraordinate concept, 3) the list of the associated key words, 4) the characteristics, 5) the examples and non-examples, 6) the space for a new example, and 7)



the concept definition. When using the Concept Mastery Routine, the most important aspect is that it is carried out interactively with the students and instructor. Using these techniques instructors can select concepts from the subject area material and prepare a specific graphic device known as a Concept Diagram. Regular students and learning disabled students showed gains in their performance tests on conceptual material when the Concept Diagrams were used (Bulgren et al., 1988).

***The Concept Comparison Routine.*** (Bulgren, Lenz, Deshler, & Schumaker, 1995; Lenz & Deshler, 2004) depicts the main elements of two or more previously studied concepts. Students and teachers together complete this table. The sections include: 1) naming the concepts, 2) naming the broader or supraordinate concept, 3) listing the like characteristics of each concept, 4) listing the unlike characteristics of each concept, 5) listing the more general category of the like characteristics, 6) listing the more general category of the unlike characteristics, 7) listing the more general category of the unlike characteristics, 8) summary, and 9) an assignment to check the understanding of the concepts. This is a routine that can be used to review and maintain continuity between concepts.

The third component of content enhancement is *procedures* (Bulgren & Lenz, 1996). For the devices and routines to be effective, the instructor must implement them into the classroom in an effective way. To do this, the instructor and student form a partnership to carry out the learning process. A necessary part of the Content Enhancement is for the instructor to inform and involve the students in the process of creating a partnership. This is achieved by making them aware of the benefits of using the Content Enhancement, as shown by their performance on tasks. In this partnership, the instructor is not merely a dispenser of the subject area content information, but is a facilitator for the students. This entails the following: 1) The instructor

must inform the students about the use of the routines and how the routines will help them learn the necessary material for the course. The students must be aware of the importance of the routines in their learning process. 2) The instructor must provide clear instruction to the students. This is done by the cue-do-review sequence. The instructor must cue the students to the necessary information and teach the content using the visual device along with the linking steps, and then the instructor must review the content and the process of the teaching routine with the students. 3) The instructor must promote interactive instruction with the students. This includes both planned and unplanned routine instruction and requires a great deal of flexibility on the part of the teacher.

This use of the routines requires the instructor to set a new goal for students in the classroom. Instead of teaching to the middle group or to the higher group, this method of teaching, Content Enhancement, is designed to respond to the needs of all students in the classroom. It is imperative that all students know a method of learning in order to succeed in today's rapidly changing society (Bulgren & Lenz, 1996).

## **Mathematics**

### **Algebra.**

Algebra is a branch of mathematics. Mathematics is built of human activity which means that mathematics is a language of human action (Confrey, 1990). Four conceptions of algebra were identified as 1) algebra as generalized arithmetic, 2) algebra as study of procedures, 3) algebra as study of relationships among quantities, and 4) algebra as study of structures. These conceptions of algebra correspond to the different ways variables are used. These uses for a variable can be 1) pattern generalizers, 2) unknowns or constants, 3) arguments or parameters, and 4) arbitrary marks on paper (Usiskin, 1988).

These conceptions of algebra, along with the different uses of variables, cause researchers to make decisions about what parts of algebra to teach and what parts of algebra to emphasize to students (Chazan, & Yerushalmy, 2003; NCTM 1989, 2000). The National Council of Teachers of Mathematics places an importance of having students know the difference in the meanings of expressions, equations and inequalities. One way they do this is to do mathematical modeling to represent quantitative relationships. Then determine the class of functions that model this relationship (NCTM, 2000).

School algebra is important in mathematics curriculum during this time of educational reform. Algebra instruction is significant for all students, not just those planning to attend college (Chazan & Yerushalmy, 2003). The National Mathematics Advisory Panel argues algebra is a major concern in the high schools (National Mathematics Advisory Panel, 2008). It has become a gateway course because students need algebra to study advanced mathematics. This in turn hinders their ability to achieve in our society today because many jobs require a study of higher mathematics (Evan, Gray, & Olchefske, 2006).

### **College Algebra.**

College Algebra is the introductory college mathematics course that prepares students for higher level mathematics. It has traditionally focused on content knowledge, which meant knowing certain pieces of the subject matter (AMATYC, 1995). A change in this approach is based on the idea that “knowing mathematics” means being able to do mathematics by using problem solving (AMATYC, 1995; Schoenfeld, 1992). Two of the themes for the AMATYC standards are 1) symbolism and algebra and 2) function. This means some traditional topics receive increased attention and some topics receive decreased attention. Decreased attention is suggested for the manipulation of complicated radical expressions, factoring, and rational

expressions. Increased attention is given to the functional approach. This includes visual representation of concepts, integration of the concept of functions through the general behavior of functions, and the connections of functional behavior with exploratory graphical analysis (AMATYC, 1995). Technology is changing the purpose of algebra from symbolism to a study of relationships. These relationships are real-world applications depicted by functions and variables that actually vary. This new vision of school algebra requires conceptual understanding along with symbol sense and mathematical modeling (Heid, 1996).

College Algebra is one branch of mathematics where the basic principles of *Beyond Crossroads* can be addressed (Haver, 2007). One of these principles is concerned with equity and access to high quality and effective instruction. This instruction should also include student support services (AMATYC, 2006). In many situations college algebra courses do not meet the guidelines of *Crossroads in Mathematics* (Haver, 2007). To meet these changing methodologies is a challenge for many faculty members (Haver, 2007). *College Algebra Guidelines*, recommended by Curriculum Renewal Across the First Two Years, provides a vision for what students should experience in a classroom (Curriculum Renewal Across the First Two Years [CRAFTY], 2006; Ganter & Barker, 2004). These guidelines not only address equity and access, but they also address using research-based practices to provide students a meaningful experience for students (Haver, 2007). To implement the standards in *Beyond Crossroads*, many stakeholders must be involved to promote student learning. Some of these stakeholders include publishers, community citizens, college and university faculty, business, and industry (Parsons, 2007).

Conceptual understanding for college algebra is more than just seeking answers for the latest test. It involves thinking and creating concepts. This is not the traditional view of

knowing mathematics, but students are achieving what the calculator cannot do (Gordon & Gordon, 2006).

### **Teaching and Learning with Understanding.**

A mathematical idea, procedure, or fact is understood if it is part of a students' internal network. This degree of understanding is determined by the number and strength of the connections. It must also be linked to existing networks with stronger and numerous connections (Hiebert & Carpenter, 1992). This is supported by other researchers' work in mathematical education (Hiebert, 1986; Hiebert & Carpenter, 1992; Janvier, 1987; Polya, 1957). To create new networks older networks must already exist. Past experiences create networks that the learner uses to create new information (Ausubel, 1968). The existing networks influence new networks that are formed. Understanding can be thought of as a process of making connections between existing networks and new information. Since learning with understanding is important, the instruction for students should be designed so students build connections. To do this, it is important that teachers also develop connected structures of knowledge about teaching (Hiebert & Carpenter, 1992).

Constructing maps and diagrams is a potential educational value because it promotes the conceptual and methodological understanding of mathematics (Afamasaga-Fuata'i, 2009). Studies show that students learn more material if it is emphasized in their mathematics classroom than the material that is not emphasized (Huntley & Rasmussen, 2002). Another author, Robinson, believes the vision of NCTM implies that students learn better when they are actively involved in thinking and doing mathematics (Robinson, 2006).

### **Textbook.**

The textbook, *College Algebra: Graphs and Models* (Bittinger, Beecher, Ellenbogen, & Penna, 1997) was chosen for this study. This textbook covers college algebra material and was appropriate for a one-term course. The approach of this textbook was more interactive and visual than most textbooks at the time it was chosen. It incorporated the use of the graphing calculator and provided a review of intermediate algebra topics to unify the diverse backgrounds of students. One useful element was the side-by-side algebraic and graphical solutions with each method, providing a complete solution. This text also emphasized functions by presenting the attributes of functions by using a function family approach. Regression or curve fitting to model data was another visual theme of this textbook (Bittinger et al., 1997). In addition, the second edition highlighted the importance of connecting concepts by using a visual approach (Bittinger, Beecher, Ellenbogen, & Penna, 2001).

### **Procedures and Concepts.**

The two main types of mathematical knowledge are procedural and conceptual. It is not important to distinguish the most dominant, but to know how they are related (Glaser, 1979; Schoenfeld, 1992). Conceptual knowledge is defined as knowledge that is understood with connected networks (Hiebert, 1986; Schoenfeld 1992). Procedural knowledge is knowledge that can be expressed as a sequence of actions. The connections for procedural knowledge are between the succeeding actions or individual steps. Examples of this are the standard symbolism of algorithms in algebra. Both of these types of knowledge are needed for mathematical expertise. The relationships between conceptual and procedural knowledge depend on the connections the learner constructs between the different representations. Theory and empirical data currently favor stressing understanding over skill proficiency. One way to connect new

concepts and procedures to new knowledge is to relate the new material with the student's prior knowledge. It is also important to provide connections to the symbolic knowledge of the procedures (Hiebert & Carpenter, 1992).

## **Overview of Content**

### **Review.**

One of the main important concepts of algebra is the variable. In most classrooms this is denoted by "x". This variable can take on many meanings in the algebra content. One of the meanings is to use "x" as a placeholder for the unknown. This is the traditional approach for understanding "x" in the algebra classroom. An additional view of "x" is that it is a value that is changing. This is the meaning that is promoted by the "functional approach" (Heid, 1996).

Sfard (1991) states the historical evolution of algebraic symbolism can be described in procedural-structural terms where algebra topics are presented in a particular order (Sfard, 1991). These include beginning with literal terms, expressions, and followed by equations (Kieran, 1992; Sfard 1991). Research suggests that this formal method resulted in an absence of structural conceptions (Kieran, 1992). To overcome some of these missing concepts the order of expressions, equations, and inequalities was kept while having students analyze the attributes of each type of procedure. The common attributes to analyze were the instructions, the placement of the equal sign, and the goal of the problem.

Keppel states that if students have to drop out of school, they should also have acquired a basic knowledge of the structure of mathematics and have known the power of mathematics (Keppel, 1963). The review of algebraic symbolism in this study was taught using a fundamental structure approach to the necessary procedures for college algebra. Problems for the review in the textbook were assigned as homework.

An expression was introduced as a combination of numbers, variables and operations. This combination represented a phrase as opposed to a statement. Then the actions for simplifying an expression included combining like terms, using order of operations, taking powers, factoring, substituting and distributing.

An equation is a statement composed of two expressions that were separated by an equals sign. Since the concept of equality is difficult for students, the concept of equivalence was emphasized. Understanding the placement of the equal sign, the meaning of balance and that the answer could precede the equal sign were some of the major benchmarks defined by Carpenter (Carpenter, Franke, & Levi, 2003). The actions for equations were the common action of, “what you do to one side, you do to the other.” This action included using the same operation, the inverse operation and powers or exponents.

An inequality is a statement composed of two expressions that were separated by an inequality sign. The action for an inequality was turning the symbol when multiplying or dividing by a negative number.

### **Graphing Calculators.**

One of the Pedagogical Standards of AMATYC and NCTM is to teach with technology (AMATYC, 1995; NCTM, 1991). Also, a major focus of the reform movement is to teach for understanding (AMATYC, 1995; NCTM, 1991). The graphing calculator addresses these two important changes for teaching algebra. The graphing calculator allows students to examine college algebra topics that were not possible in previous classrooms. The five major parts of the calculator for college algebra included: 1) home screen, 2) graphing and windows, 3) function calculations, 4) tables, and 5) regression. Each of these parts is directly related to the major topic, functions, in college algebra. Each part not only concentrated on the major screens of the



calculator, but also enhanced the students' ability to view the topics of college algebra. The graphing calculator allowed students to study 1) the different representations of functions and how they are related, 2) the translations and transformations, and 3) applications including both procedures such as finding equations of lines and the modeling of data using regression.

### **Function.**

A curriculum proposal for secondary schools proposed that precise vocabulary, including the use of functions, prepared students to grasp the necessary concepts for further study of mathematics (Hovis et al., 2003; Keppel, 1963). Mathematics teaching should include interactions between students and teachers so that students can develop sufficient confidence to understand the power of mathematics. This is accomplished by the use of the mathematical language of graphs, tables, and symbolic algebra in the study of various functions (Swan 1985; Yerushalmy & Schwartz, 1993). The concept of functions is important in the study of mathematics for secondary students because it is a unifying theme of mathematics. This special correspondence between variables is a concept that represents many situations in the real world (Dossey, 1998; Hovis et al., 2003). The standard, functions, provides a guide for mathematics reform for teaching secondary school students (NCTM, 1989). This plan for reform is further emphasized in a later standard, learning environments. It includes providing an environment in which students gain mathematical power by strong communication between the community of learners, students and teachers (NCTM, 1991). *Crossroads in Mathematics* (AMATYC, 1995) also included functions as one of its standards for content. They suggested topics for teaching to include generalizations about the function families, including linear and quadratic functions. One of the standards for pedagogy included using multiple representations to help students solve problems. The guidelines for content included increasing attention to integrating the concept of

functions within the course and analyzing the general behavior of functions using the graphing calculator (AMATYC, 1995). Using the calculator to model real data was also encouraged. In conclusion, “The concept of function is a central one in mathematics” (Dossey, 1998; Hovis et al., 2003).

### **Teaching for Understanding.**

According to Hiebert (1997) instruction in classrooms that focuses on understanding has some core features such as 1) the nature of classroom tasks, 2) role of the instructor, 3) social culture of the classroom, 4) mathematical tools, and 5) equity and accessibility (Hiebert, 1997). These core principles were followed in the experimental and control classrooms. They included connecting with individual students, sharing essential information, valuing each student’s participation, and making tasks available to every student. These values were additionally enhanced by using the Content Enhancement Routines in the experimental classrooms.

“A Vision for School Mathematics” implies that students learn best when they are involved in doing mathematics (NCTM, 1991, 2000). Students are not usually involved in the traditional classroom with lecture as the main format (Huntley & Rasmussen, 2002). This traditional format, lecture, usually requires students to use the lowest level of Bloom’s taxonomy (Bloom, 1984; Hovis et al., 2003). This lowest level of knowledge usually involves memorization. The use of Content Enhancement Routines emphasizes an interaction of dialogue between the instructor and the students (Bulgren & Lenz, 1996).

## **Chapter III**

### **Methodology**

#### **Introduction**

The primary goal of this study was to test algebra achievement when Content Enhancement Routines designed at the Center for Research on Learning at the University of Kansas were used with students from a mid-western community college. The chosen classes were six sections of college algebra. This study was conducted in three semesters with an experimental and control group each semester. The researcher received permission to conduct the study from the mathematics dean. The secondary goal of this study was to test the mathematics confidence scores of this group of students. Also, a correlation coefficient was used for the students' perceived value of the routines and the algebra achievement.

The methodology used to test the research questions is listed in this chapter. The following is a list of the individual sections: 1) introduction, 2) design of the experiment, 3) selection of the subjects, 4) instrumentation, 5) data collection, 6) data analysis, and 7) summary.

College Algebra is the standard introductory required mathematics course for students planning to transfer to a four-year college or university. The prerequisite for this course is intermediate algebra or a minimum of two years of high school algebra. The course consisted of the conventional topics in college algebra: a review of symbolism followed by the study of functions including linear, quadratic, absolute value, polynomial, rational, exponential and logarithmic. The college algebra sections were taught by a full-time faculty member in the department of mathematics, who was also the researcher for this study.

## Design of the Experiment

The research design was a quasi-experimental group design. It included a pretest-posttest design with an experimental and control group. An analysis of variance on the means on algebra achievement posttest scores was conducted. Also, an analysis of variance was conducted on the means on mathematics confidence posttest scores.

The quasi-experimental pretest posttest design for this study was diagrammed as follows:  
It was a design for both the algebra achievement and mathematics confidence studies:

Table #1

		Pretest		Posttest
Control	R	O1	X1 (Traditional) X2	O2
Experimental	R	O1	(Content Enhancement)	O2

Table #1 shows the quasi-experimental pretest posttest design for this study. In this design R indicates the random assignment of classes to treatment and control groups, X1 represents the control, X2 represents the experimental treatment, O1 represents the achievement/confidence pretest and O2 represents the achievement/confidence posttest (Campbell, 1963).

## Selection of Subjects

### Population and Sample.

The sample for this study was taken at a small mid-western combination urban and rural community college during three semesters. This sample included students enrolled in college algebra. The sample of students for this study was forty-seven female and forty-seven male. The majority of the students in this study attended on a part-time basis and held part-time jobs.

Most of the students were recent high school graduates between the ages of 18 and 22. Six sections of the course were involved in the study. Two sections were studied each semester. The control groups had forty-five students and the experimental groups had forty-nine students. The study was conducted in each of these six sections of college algebra. Participation in the classroom and completion of the tests were required in all sections of the course. The students followed the guidelines of the college for placement in college algebra.

### **Random Assignment of Treatments.**

The students selected their classes according to their own scheduling needs. The selections of the sections were individual choices. However, the placement procedures of the college served as a guideline for each student. Placement in college algebra included an ACT Asset score, an ACCUPLACER score, a departmental pre-test score, or an analysis of the mathematical background of the student. Each section of college algebra involved in this study met for fifty minutes each day. Since the students could not be randomly placed in the classes, the sections were chosen at random for the control and experimental groups for each of the three semesters.

### **Control Group.**

The three sections of college algebra that comprised the control group were not given any exposure to any of The Content Enhancement Routines. The control group received the traditional method of instruction, which included following the sequence of the textbook using predominately lecture format. Because the students sat in pairs at tables, there was some cooperative group work both during the lecture and during the time students worked on assignments.

### **The Experimental Group.**

The students in the experimental group received The Content Enhancement instruction from the researcher. To prepare to use The Content Enhancement Routines, the researcher analyzed the content of the course, organized the information into meaningful units, identified the major concepts and drafted the routines for the study. The students in the experimental group were instructed using a combination of the following routines: 1) The Course Organizer, 2) The Unit Organizer, 3) The Lesson Organizer, 4) The Concept Anchoring, 5) The Concept Mastery, and 6) The Concept Comparison.

Advance organizers were used to help the student recognize the organization and structure of the material. The organizer routines in The Content Enhancement Series were The Course Organizer, The Unit Organizer, and The Lesson Organizer. These routines were partially completed by the teacher before the class because too much class time would be necessary for students to write everything, especially the self-test questions. The students completed most of the organizer routines in class. There was an emphasis by the instructor on the maps for each organizer routine.

Once the students were aware of the organization and structure, the instructor incorporated three concept routines to help the students master the content. The three concept routines included The Concept Diagram, The Concept Anchoring, and the Concept Comparison. The Concept Anchoring was used for the concept of function. The Concept Mastery was used for the concepts of function, domain, and slope. The Concept Comparison was used for linear and quadratic functions.

The students were instructed in the routine procedures according to the guidelines by the authors. The general guidelines suggested using the routines explicitly by having the students

follow the device closely, explain how the device enhances learning, use the device actively, and participate by taking notes and asking questions.

The researcher, in a partnership with the students, used the Cue-Do-Review Sequence and the Linking Steps for each device. The Cue component referred to the following steps: 1) Cue students to the graphic, 2) Remind the students of the benefits, and 3) Prompt students to take notes and help construct the graphic. The Do component referred to 1) developing each graphic device using the unique appropriate Linking Steps, embedded in each device and 2) creating a partnership with the students by constructing the graphic together. The Review component referred to the following steps: 1) Review the content in the graphic, and 2) Review the student understanding of the thinking processes involved in each routine (Bulgren et al., 1995; Bulgren et al., 1993, 1994; Lenz et al., 1994; Lenz et al., 1993; Lenz et al., 1998). These steps were practiced by the instructor as the study progressed. Since this new instructional technique required practice and thoughtful reflection, the instructor revised her technique as the semesters progressed. The students were paired at tables and often consulted with each other before offering their viewpoints to the entire class. Although the experimental group had homework problems similar to those of the control group, the ordering of the problems was different due to the restructuring of the content for the experimental group.

### **Textbook and Resource Center.**

The textbook was the same for all sections in the experimental and control groups. It was *College Algebra: Graphs and Models* (Bittinger, Beecher, Penna, & Erlenbaum, 1997). One semester the group had a newer edition of the book (Bittinger, Beecher, Penna, & Erlenbaum, 2003). However, most sections of the book were nearly identical. Students in both sections every semester took the same tests and had the same homework assignments from the textbook.

All students in both the control and experimental groups were able to go to a resource center for tutoring. They had the opportunity to sign up for peer tutoring or to receive tutoring from faculty members. All students in the study had equal access to the resource center options.

### **Instrumentation**

The instruments used in this study were the algebra achievement test, the mathematics confidence test, and the attitude survey. The algebra achievement test was constructed from the test generator that was designed as a supplement to the textbook for the instructor to use. The mathematics confidence test was the Confidence in Learning Mathematics Scale from the collection of Fennema-Sherman Mathematics Attitudes Scales (Fennema & Sherman, 1976). The attitude survey on students' perceived value of The Content Enhancement Series was constructed from a model survey (Palagallo & Blue, 1999) by the researcher.

#### **Algebra Achievement Exam.**

The pretest and posttest algebra achievement test was designed using the test generator. It was one of the supplements that accompanied the textbook for the instructors from the publishing company of the textbook. The algebra achievement test consisted of thirty-three multiple choice questions with four choices for each answer. The students were told to mark E or the fifth choice if they did not agree with any of the four choices. The selection of test questions was made by the researcher. Two mathematics faculty members approved the questions as being appropriate for college algebra curriculum. This test covered linear and quadratic functions. A summary of the questions:

Linear/Quadratic Differences	4 questions
Domain	5 question
Slope	5 questions



Function Attributes	10 questions
Function Notation	2 questions
Inequalities	2 questions
Equations	2 questions
Regression	3 questions

The questions were a combination of topics covering function notation, regression, translations, symmetry, even/odd functions, zeroes, maximum/minimum, inequalities, distance, midpoint, applications and graph analysis.

In designing the algebra achievement test, the researcher focused on questions that were conceptual in nature with a minimum amount of algebraic manipulation or tedious procedures. The focus was on acquiring concepts and understanding their relationships. The test included questions with simple procedures that focused on concepts. The four questions related to the concepts of linear and quadratic functions emphasized the differences between the two types of functions. The five questions related to the concept of domain emphasized the characteristics and examples of domain. The five questions related to the concept of slope are questions that emphasized the characteristics and examples of finding slope. The other questions were general concept types of questions that would be included in the study of linear and quadratic functions, the two major types of functions in this study (see Appendix D: Instruments).

### **Confidence in Learning Mathematics Scale.**

The Fennema-Sherman Mathematics Attitudes Scales was a set of scales that measured Usefulness of Mathematics, Confidence in Learning Mathematics, Mathematics Anxiety and Mathematics as a Male Domain. This study was concerned only with the Confidence in Learning Mathematics Scale. This scale is a 12-item Likert scale consisting of six positively

stated items and six negatively stated items. Each item had five choices for responding: strongly agree, agree, undecided, disagree, and strongly disagree. Each response was given a score from 1 to 5 with the higher score being given to the more favorable response, which means having a positive effect on the learning of mathematics. The Confidence in Learning Mathematics had a split-half reliability of .93 (Fennema & Sherman, 1976).

The Confidence in Learning Mathematics Scale was designed to measure the confidence in one's ability to learn mathematics and to perform mathematical tasks. The range of this confidence was from a distinct lack of confidence to definite confidence. Mathematics anxiety, confusion, interest, enjoyment and zest for problem solving were not attributes of this scale (see Appendix D: Instruments).

#### **Students' Perceived Value of The Content Enhancement Series.**

For one of the semesters the researcher decided to measure the students' perceived value of The Content Enhancement Series in helping them learn college algebra concepts by giving a survey. The survey was given for fifteen concept or organizer routines. The surveys were given after the end of each of the three units. This survey was designed and modeled after a survey used at the University of Akron in Ohio (Palagallo & Blue, 1999). Four choices were included in the perceived value survey. The student choices were: 1) extremely helpful, 2) moderately helpful, 3) slightly helpful, and 4) not helpful. Students were asked to rate each routine with respect to its value in learning the college algebra concepts. The means of each students' scores were calculated by the rating of 4 – extremely helpful; 3 moderately helpful; 2 – slightly helpful; and 1 – not helpful (see Appendix D: Instruments).

### **Reliability of Instruments.**

The reliability of each of the three testing instruments was determined by using Cronbach's Alpha. The three pretest instruments were algebra achievement, mathematics confidence, and students' perceived value. The reliability for each of the three pretest instruments was greater than .802. The reliability of the three pretest instruments were computed as: 1) algebra achievement .802, 2) mathematics confidence .946, and 3) student's perceived value .848. The reliability of the three sub-topic pretests computed as: 1) linear and quadratic .426, 2) domain .391 and 3) slope .629.

### **Concept Routines.**

The Concept Routines included the Concept Mastery, Concept Comparison, and the Concept Anchoring. These routines were co-constructed by the instructor and the students during the class. The concept devices were given to each student at the beginning of the class while the instructor informed the class that the routine would help them learn the necessary material. A verbal interaction between the instructor and the students followed and was often initiated with questions from the instructor. The instructor prepared a sample copy prior to the class to have a supply of questions for students. This assured all important parts of the chosen concept were covered and recorded by students. During the discourse the instructor and students completed the routines interactively. This was followed by a review of the concept routine. This cue-do-review was an important part of the teaching routine. There is a slight difference in each concept routine between the semesters because each section of students was different. Also, time allowances for each part of the concept routine differed between sections. The concept routines in the appendix include the major sections completed by the instructor and students in each class.

### **Organizer Routines.**

The Organizer Routines included the Course, Unit, and Lesson. These organizers were co-constructed by the instructor and the students during the class. The questions for each of the organizers were prepared by the instructor while planning the content. This planning process included determining the major outcomes for the course, each unit, and each lesson. This reflective process changed slightly from semester to semester after teaching and receiving feedback from the students. The questions were printed on the organizers before distribution to the students. This was done because of the length of the class sections. The questions also initiated further verbal interaction from the students. The remaining parts of the organizers were completed interactively during the class section. The students made notes on the organizers under each of the “bubbles” or pieces of information. The visual maps on all the organizers helped students see connections of concepts and relationships. There is a slight difference in each organizer routine between semesters because each section of students was different. Also, time allowances for each part of the organizer routine differed between sections. The organizer routines in the appendix include the major sections completed in each class by the instructor and students.

### **Data Collection**

This study was conducted during each of the three semesters. The Algebra Achievement Test and the Confidence in Learning Mathematics Scale was given during the first week of class to the experimental and control groups. This allowed time for students who had to do a drop/add enrollment change. It was given during the scheduled class. If a student was absent, he or she took it during the following scheduled class. These two instruments were also given as a class exam at the end of the study. The study concluded after the unit of quadratic functions. Before

this posttest exam, the students received a review during the previously scheduled class. The students' Perceived Value Scale of The Content Enhancement Routines was given at the end of each unit of study on the experimental group for one semester of the study.

## **Data Analysis**

### **Hypotheses.**

For hypothesis one, there will be a significant positive correlation between the means on the mathematics confidence pretest scores and the algebra achievement posttest scores. For hypothesis two, the experimental group will have higher means on algebra achievement posttest scores than the control group. The experimental group will have higher means on mathematics confidence posttest scores than the control group.

For hypothesis three, the experimental group will have higher means on algebra achievement posttest scores on linear and quadratic questions than the control group. For hypothesis four, the experimental group will have higher means on algebra achievement posttest scores on domain questions than the control group. For hypothesis five, the experimental group will have higher means on algebra achievement posttest scores on the slope questions than the control group.

For hypothesis six, there will be a significant positive correlation between the means on the students' perceived value scores of The Content Enhancement Series and the algebra achievement posttest scores.

Table #2

### **Summary of Tests**

Hypothesis	Independent Variables	Dependent Variables
1. Correlation	Confidence (pre)	Algebra Achievement (post)
2. Two-way ANOVA	Group Gender	Algebra Achievement (post)

Two-way ANOVA	Group Gender	Mathematics Confidence (post)
3. One-way ANOVA	Group	Linear & Quadratic Achievement (post)
4. One-way ANOVA	Group	Domain Achievement (post)
5. One-way ANOVA	Group	Slope Achievement (post)
6. Correlation	Perceived Value	Algebra Achievement (post)

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Table #2 is a summary table of the tests that were used for the six hypotheses.

Hypothesis one and six were correlations. Hypothesis two used two-way ANOVA. Hypotheses three, four, and five used one-way ANOVA's.

### **Summary**

The Content Enhancement Model is an instructional model that instructors use to help students acquire content information by helping them identify, organize, comprehend, and memorize critical content information. It uses graphic advance organizers, visual graphics, and planned interaction between the teacher and the students. Ninety-four community college students, studying college algebra, and the researcher participated in this study to determine the effects of the use of The Content Enhancement Model on algebra achievement and confidence level of ability to do mathematical tasks, and on perceived value of The Content Enhancement Routines. These effects were determined by two correlations: 1) on the mathematics confidence pretest scores and the algebra achievement posttest scores, and 2) of the students' perceived value scores and the algebra achievement posttest scores. Also, the effects of the means on the algebra achievement posttest scores of the gender and group, and the effects of the means on mathematics confidence posttest scores of the gender and group were studied using the analysis of variance statistical test.

## **Chapter IV**

### **Presentation and Analysis of Data**

#### **Introduction**

The purpose of this study was to investigate the effects of the use of The Content Enhancement Model on college algebra students' achievement and mathematics confidence scores and to determine the effects of student's perceived value of The Content Enhancement Routines on their algebra achievement. This was accomplished by examining the algebra achievement posttest scores and mathematics confidence posttest scores of experimental and control groups along with the gender factor. It also examined the perceived value of The Content Enhancement Routines by the students. This chapter presents the results of the data analysis for the six research questions.

#### **Descriptive Statistics**

The SPSS program was used to analyze the student data. This included the algebra achievement pre- and posttest scores and the mathematics confidence pre- and posttest scores. It also included the algebra achievement posttest scores for each of the sub-topics including linear/quadratic functions, domain, and slope. The student algebra achievement posttest scores were measured by the total number of correct problems out of a possible total of 33. This was done for the experimental and control groups as well as for the gender.

Data for mathematics confidence scores were collected for each student. The student mathematics confidence scores were measured by the total number of points on a twelve question Likert Scale. This was a scale from 1 to 5 with 5 being a positive response to mathematics. This was done for the experimental and control groups and disaggregated by the gender groups.

The students' perceived value of using The Content Enhancement Routines/Organizers was also collected. The student's perceived value scores were determined by calculating the means of the points for fifteen of The Content Enhancement Routines. This was a Likert Scale from 1 to 4 with 4 being a positive response to the Content Enhancement Routines. This was done only for the experimental group in one semester.

Table #3

Mean Scores of Gender and Group

		Females (N)	Males (N)	Control (N)	Experimental (N)
Achievement	Pre	.3327 (47)	.4262 (47)	.3569 (45)	.4001 (49)
	Post	.6905 (47)	.6157 (47)	.6020 (45)	.7001 (49)
Linear-Quadratic	Pre	.3351 (47)	.4255 (47)	.3556 (45)	.4031 (49)
	Post	.7500 (47)	.6862 (47)	.6722 (45)	.7602 (49)
Domain	Pre	.3574 (47)	.4468 (47)	.3289 (45)	.4694 (49)
	Post	.7234 (47)	.6383 (47)	.6400 (45)	.7184 (49)
Slope	Pre	.4809 (47)	.6000 (47)	.5289 (45)	.5510 (49)
	Post	.7234 (47)	.6511 (47)	.6178 (45)	.7510 (49)
Confidence	Pre	3.3862 (41)	3.9123 (38)	3.5365 (32)	3.7092 (47)
	Post	3.2663 (46)	3.8739 (39)	3.4342 (38)	3.6348 (47)

Table #3 shows the mean scores of gender and group. The gender columns include the female and male scores which are the combination of the scores of the experimental and control groups. The group columns include the control and experimental scores which are the combination of the scores of the female and male groups. The information for overall algebra achievement scores is graphed in the following Figure #1.

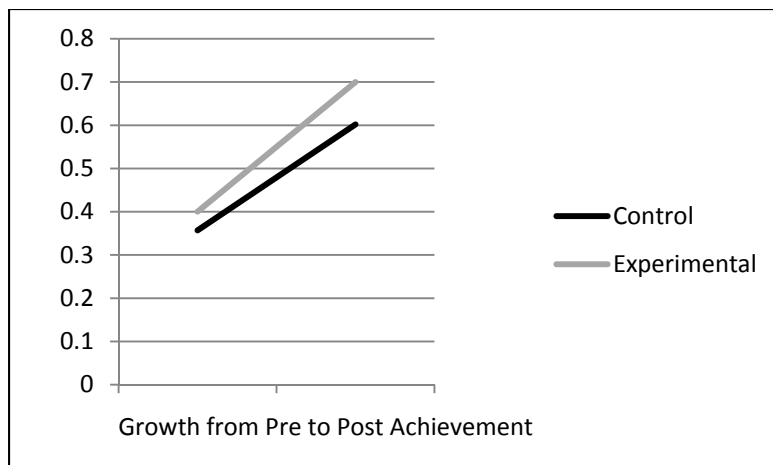


Table #4

Mean Scores by Gender and Group					
		Pre-Achievement (N)	Post-Achievement (N)	Pre-Confidence (N)	Post-Confidence (N)
Females	Control	.2948 (22)	.6446 (22)	3.0156 (16)	2.9444 (21)
	Experimental	.3661 (25)	.7309 (25)	3.6007 (24)	3.5367 (25)
Males	Control	.4163 (23)	.5613 (23)	4.0972 (12)	4.0392 (17)
	Experimental	.4356 (24)	.6679 (24)	3.7778 (21)	3.7462 (22)

Table # 4 shows the means of the pre- and post-test achievement scores and the pre and post- test confidence scores of the females and males. The female and male groups were subdivided to show the experimental and control groups' means.

Figure #1



### Reliability of Instruments.

The reliability of each testing instrument was determined by using the correlations of Cronbach's Alpha. This data was programmed in SPSS and yielded the outcomes of the reliability for the five instruments, listed in Table #5.

Table #5

Internal Consistency Reliability Coefficients					
	N items	N participants	Mean	SD	Cronbach's $\alpha$
Achievement Pre-test	33	94	.379	.176	.802
Linear & Quadratic	4	94	.380	.145	.426
Domain	5	94	.402	.118	.391
Slope	5	94	.483	.100	.629
Achievement Post-test	33	94	.653	.138	.920
Confidence Pre-test	12	79	3.639	.324	.946
Confidence Post-test	12	85	3.545	.279	.940
Perceived Value	15	11	2.978	.259	.848

## Results of Testing the Research Questions

### Research Question 1.

Is there a correlation between the means on the mathematics confidence pretest scores and algebra achievement posttest scores?

Results of a Pearson correlation indicate there is not a statistically significant relationship between mathematics confidence pretest scores and algebra achievement posttest scores ( $r = .159, p > .05$ ).

### Research Question 2.

Is there a difference in the means on algebra achievement posttest scores for students in the experimental and control groups assuming no prior differences? Is there a difference in the means on mathematics confidence posttest scores for students in the experimental and control groups assuming no prior differences? The means for the groups are averaged across male and female students.

One 2-way ANOVA was conducted on algebra achievement posttest scores across gender and group. Results indicate there is not a significant main effect for gender. However, results for

a main effect for group indicate a statistically significant difference in favor of the experimental group using an alpha level of .05:  $F(1,90) = 3.817, p \leq .05$ .

One 2-way ANOVA was conducted on mathematics confidence posttest scores across gender and group. Results show there is no main effect for group. However, results for a main effect for gender indicate a statistically significant difference in favor of the males:  $F(1,81) = 11.888, p = .001$ .

### **Research Question 3.**

Is there a difference in the means on the algebra achievement posttest scores on the linear and quadratic questions between the control and experimental groups?

Group differences were assessed using an analysis of variance (ANOVA) statistical test. The result of the ANOVA is not statistically significant. There is no statistically significant difference in the means on the algebra achievement posttest scores on the linear and quadratic questions ( $F(1,92) = 2.219, p = .15$ ) between the control and experimental groups.

### **Research Question 4.**

Is there a difference in the means on the algebra achievement posttest scores on the domain questions between the control and experimental groups?

Group differences were assessed using analysis of variance (ANOVA) statistical test. The result of the ANOVA is not statistically significant. There is no statistically significant difference in the means on the algebra achievement posttest scores on the domain questions ( $F(1,92) = 1.851, p = .18$ ) between the control and experimental groups.

### **Research Question 5.**

Is there a difference in the means on the algebra achievement posttest scores on the slope questions between the control and experimental groups?

Group differences were assessed using analysis of variance (ANOVA) statistical test. The result of the ANOVA is statistically significant. There is a statistically significant difference in the means on the algebra achievement posttest scores on the slope questions ( $F(1,92) = 4.679$ ,  $p = .03$ ) between the control and experimental groups.

**Research Question 6.**

Is there a correlation between the means on the students' perceived value scores and the algebra achievement posttest scores?

Results of a Pearson correlation indicate there is not a significant relationship between perceived value scores and the algebra achievement posttest scores ( $r = -.07$ ,  $p > .05$ ). The results of the survey indicate a positive effect. Eighty-two percent of the students reported The Content Enhancement Routines were moderately or extremely helpful with forty-five percent reported The Content Enhancement Routines were extremely helpful.

## **Chapter V**

### **Summary, Discussion, and Conclusions**

#### **Introduction**

This chapter gives a summary of the research study with the conclusions related to the six research questions. This is followed by a discussion of the findings with some of the limitations of the study. These findings lead to implications for practice and recommendations for future research. A final conclusion is included.

#### **Summary of the Analysis**

For question 1, the results of a Pearson correlation indicate there is not a significant relationship between mathematics confidence pretest scores and mathematics achievement posttest scores ( $r = .159, p > .05$ ).

For question 2, one 2-way ANOVA was conducted on algebra achievement posttest scores across gender and group. Results indicate there is not a significant main effect for gender. However, results for a main effect for group indicate a significant difference in favor of the experimental group using an alpha level of .05:  $F(1,90) = 3.817, p \leq .05$ .

One 2-way ANOVA was conducted on mathematics confidence posttest scores across gender and group. Results show there is no main effect for group. There is a significant main effect for gender in favor of the males:  $F(1,81) = 11.888, p = .001$

For question 3, group differences were assessed using an analysis of variance (ANOVA) statistical test. The result of the ANOVA test is not statistically significant. There is no statistically significant difference in the means on algebra achievement posttest scores on the linear and quadratic questions ( $F(1,92) = 2.219, p = .15$ ) between the experimental and control groups.

For question 4, group differences were assessed using analysis of variance (ANOVA) statistical test. The result of the ANOVA is not statistically significant. There is no statistically significant difference in the means on algebra achievement posttest scores on the domain questions ( $F(1,92) = 1.851, p = .18$ ) between the experimental and control groups.

For question 5, group differences were assessed using analysis of variance (ANOVA) statistical test. The result of the ANOVA is statistically significant. There is a statistically significant difference in the means on algebra achievement posttest scores on the slope questions ( $F(1,92) = 4.679, p = .03$ ) between the experimental and control groups in favor of the experimental group.

For question 6, the results of a Pearson correlation indicate there is not a significant relationship between perceived value scores and the algebra achievement posttest scores ( $r = -.07, p > .05$ ). The results of the survey indicate a positive effect. Eighty-two percent of the students reported The Content Enhancement Routines were moderately or extremely helpful with forty-five percent reported The Content Enhancement Routines were extremely helpful.

This study supported the research in Chapter II. This research indicated that The Content Enhancement Routines increased achievement in the college algebra content in favor of the experimental group. Since the Standards of NCTM and AMATYC advocate the teaching of concepts in algebra as well as necessary procedures, this method using The Content Enhancement Routines was an effective way to increase college algebra achievement in the community college setting.

### **Discussion of the Findings**

The subjects in the experimental group showed overall positive results from using the Content Enhancement Routines in their college algebra classes. The use of the Concept Mastery,

Concept Comparison, and Concept Anchoring Routines for the concepts of function, domain, slope, linear and quadratic functions proved effective on the overall algebra achievement. The Course, Unit, and Lesson Organizers also contributed to the overall algebra achievement. This study showed a statistically significant difference in the means on the algebra achievement posttest scores of the group factor in favor of the experimental group. This significance showed the method of using The Content Enhancement Routines had a positive impact on the algebra achievement of subjects in the experimental group. The results of the survey indicate a positive effect. Eighty-two percent of the students reported The Content Enhancement Routines were moderately or extremely helpful with forty-five percent reported The Content Enhancement Routines were extremely helpful.

This was especially important because these college algebra basic functions form the foundation for the study of higher mathematics. A good foundation of functions is essential for the study of calculus (Hovis et al., 2003). Another necessary topic for the study of calculus is an understanding of the concept of slope (Hovis et al., 2003). For the experimental group this study showed a statistically significant difference in the means on the algebra achievement posttest scores concerning the sub-topic, slope. This is important because the foundation for the concept of a derivative in the study of calculus is the concept of slope (Hovis et al., 2003).

The sub-topics of linear and quadratic functions and domain did not show significant results from using The Content Enhancement Routines. Low reliability scores of these two sub-topics may have influenced this outcome. The study did not show a statistical significant difference in the means on the mathematics confidence posttest scores between the experimental and control groups. The perceived value of The Content Enhancement Routines by the subjects did not show a correlation on the algebra achievement posttest scores. The major reason for this

outcome was the small number of subjects in this group and the fact that two subjects already knew the material. Their comments on the survey may have been negative because they did not need the instruction, only the college algebra credits.

Overall, many students gave positive remarks about The Content Enhancement Routines and used them during the test review session. The researcher noticed the female subjects were more interested in The Content Enhancement Routines than the male subjects in the review session before the algebra achievement posttest. In fact, the female subjects were disappointed that they were not allowed to take the routines from the classroom.

### **Limitations of the Study**

There were three areas of the study that may have some limitations. One of these areas is the expertise of the researcher in the use of The Content Enhancement Routines. The researcher improved in the techniques of delivery in each of the semesters of the study. Ideally, the researcher should have had training with other algebra instructors, but this was not possible. Also, because of the time required to write the routines and organize the content, input from another instructor would have been helpful.

A second area of limitation was the number of ways a student could enroll in the college algebra classes. The tests and recommendations for enrollment were not consistent from semester to semester. It would have been more desirable if students were consistently identified for the study. This continues to be a difficulty in community colleges.

The third area of limitation was the number of questions in the sub-topics. The sub-topics of linear/quadratic and domain had only four or five questions in each group. This small number of questions may have contributed to the low reliability factors for each sub-topic. This



low reliability may have influenced the outcome of the algebra achievement posttest scores of the experimental and control groups.

### **Implications for Practice**

This research study implies that the recommendations in the Standards of NCTM and AMATYC can be met by implementing a system that uses The Content Enhancement Routines for the college algebra classroom in a community college setting. This gives educators a tool for changing the classroom from dispensing information through lecture to a classroom of interactive exchanges between students and instructors. This will help students see the “big picture” by focusing and learning the necessary concepts to achieve algebraic understanding rather than “rote memorization” of material.

The Content Enhancement Model is an instructional model that instructors can use to help students acquire the content information. This is achieved by helping students identify, organize, comprehend and memorize critical content information in the community college setting. The setting of the community college includes transfer students to four-year colleges or universities, students in two-year degree programs, students in the vocational and technology programs, and students in the community education environment, including GED students. Students in the community college setting may have the potential to benefit from instruction using The Content Enhancement Routines.

### **Recommendations for Further Research**

This study focused on the algebra achievement and the mathematics confidence levels of college algebra subjects using The Content Enhancement Routines. Subjects in the experimental and control groups were tested and surveyed. A continuation of this study is recommended for future study with a larger number of subjects. This methodology of The Content Enhancement

Routines supports the major themes of the NCTM and AMATYC standards for college algebra classrooms.

Future studies should also include a follow-up of students who study higher mathematics including calculus. A study of subjects taking calculus is a natural sequence of this study to observe their understanding of the concepts of calculus. A calculus study would increase our knowledge of how students understand the concepts of continuity, limits, and derivatives.

Future studies should also include using The Content Enhancement Routines in college developmental classes such as elementary and intermediate algebra. A thorough understanding of the concepts of expressions, equations and inequalities would give students a better foundation for college algebra and higher mathematics.

Another Concept Enhancement Routine, The Question Exploration, could be a future study for the NCTM and AMATYC Standard of problem-solving. Testing this routine would study students' ability to do algebra problem-solving skills in an organized fashion. Overall, this methodology of using The Content Enhancement Routines could also be useful in testing the content of other disciplines in a community college curriculum.

## **Conclusions**

The significance of the results of this study showed that The Content Enhancement Routines were a significant factor for teaching some algebra concepts. This study helps support the NCTM and AMATYC beliefs that “algebra is for everyone” (NCTM, 1990).

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## Appendix A: Concept Routines

### Anchoring Table

Unit: \_\_\_\_\_ Name: \_\_\_\_\_ Date: \_\_\_\_\_

Anchoring Table		
<p>② Known Concept</p> <p>neighborhood</p>		<p>① New Concept</p> <p>functions</p>
<p>④ Characteristics of Known Concept</p> <p>has several families</p> <p>different family members</p> <p>consists of different names</p> <p>houses are different</p> <p>different house numbers</p>	<p>⑥ Characteristics Shared</p> <p>common groups</p> <p>individual parts</p> <p>identification</p> <p>shapes</p> <p>location</p>	<p>⑤ Characteristics of New Concept</p> <p>has several families</p> <p>different ordered pairs</p> <p>several names</p> <p>linear or nonlinear</p> <p>different equations</p>
<p>⑦ Understanding of the New Concept:</p> <p>Functions are made up of several families. Functions have ordered pairs and are identified by their names. Functions may be linear or non-linear shapes. Each function has a different equation and is in a different location on a graph.</p>		

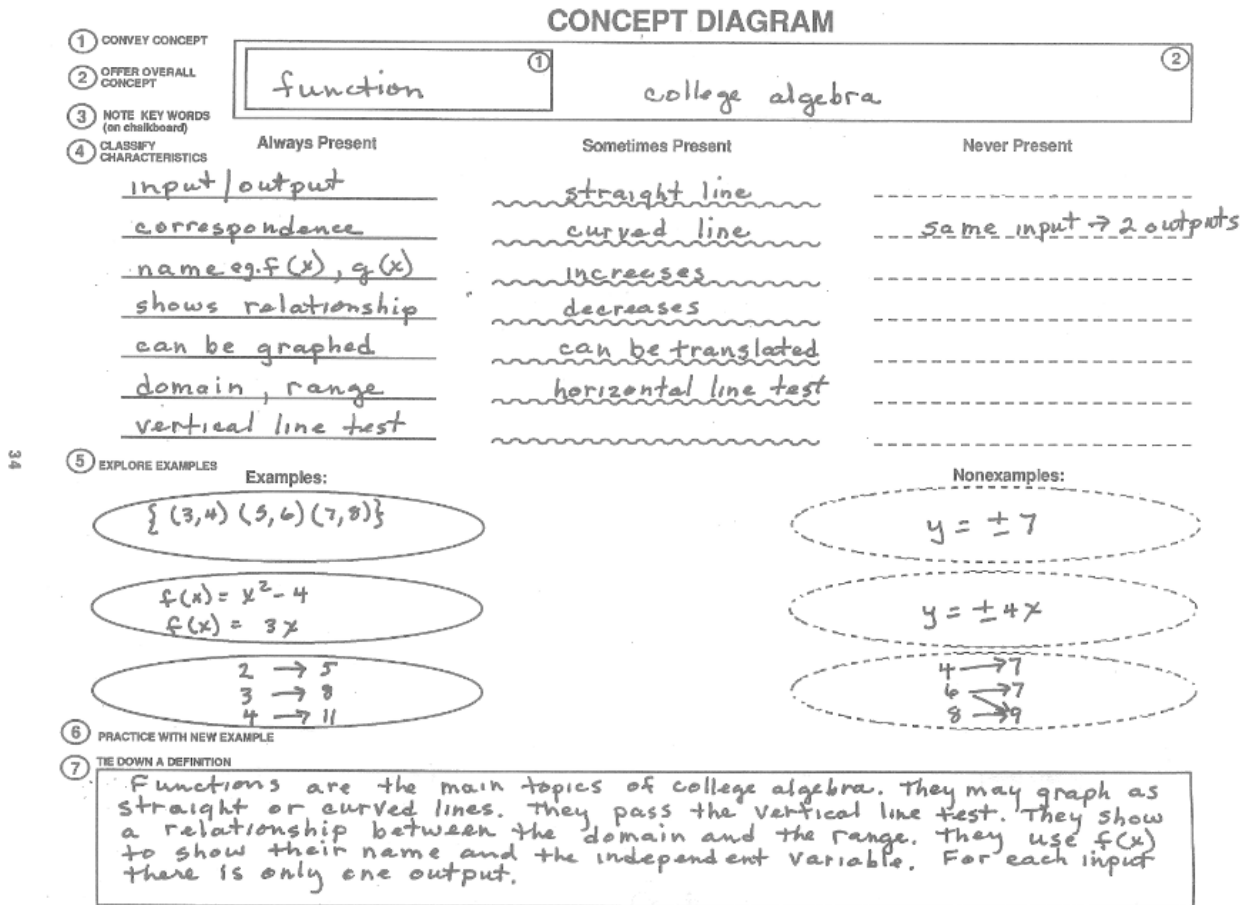
ANCHORS Linking Steps:

1 Announce the New Concept    2 Name Known Concept    3 Collect Known Information    4 Highlight Characteristics of Known Concept    5 Observe Characteristics of New Concept    6 Reveal Characteristics Shared    7 State Understanding of New Concept

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## Appendix A: Concept Routines

### Concept Diagram: Function



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## Appendix A: Concept Routines

### Concept Diagram: Domain

**CONCEPT DIAGRAM**

① CONVEY CONCEPT

② OFFER OVERALL CONCEPT

③ NOTE KEY WORDS (on chalkboard)

④ CLASSIFY CHARACTERISTICS

domain ①

ordered pairs ②

Always Present	Sometimes Present	Never Present
set of all inputs		set of all outputs
independent variables	variable	dependent variables
x-values	number	y-values
first member of pairs	set of words	second member of pairs
associated with the horizontal axis	graph	range

⑤ EXPLORE EXAMPLES

Examples:

all real numbers

$\{(3, 7), (5, 8)\}$   
 $\uparrow \quad \uparrow$

domain  $\begin{cases} 1 \rightarrow 7 \\ 2 \rightarrow 8 \\ 3 \rightarrow 9 \end{cases}$

Nonexamples:

$\begin{matrix} 2 \rightarrow 8 \\ 4 \rightarrow 10 \\ 6 \rightarrow 12 \end{matrix} \}$  range

$\{(4, 5), (5, 6)\}$   
 $\uparrow \quad \uparrow$

$y \geq 0$

⑥ PRACTICE WITH NEW EXAMPLE

⑦ TIE DOWN A DEFINITION  $x \geq 0$

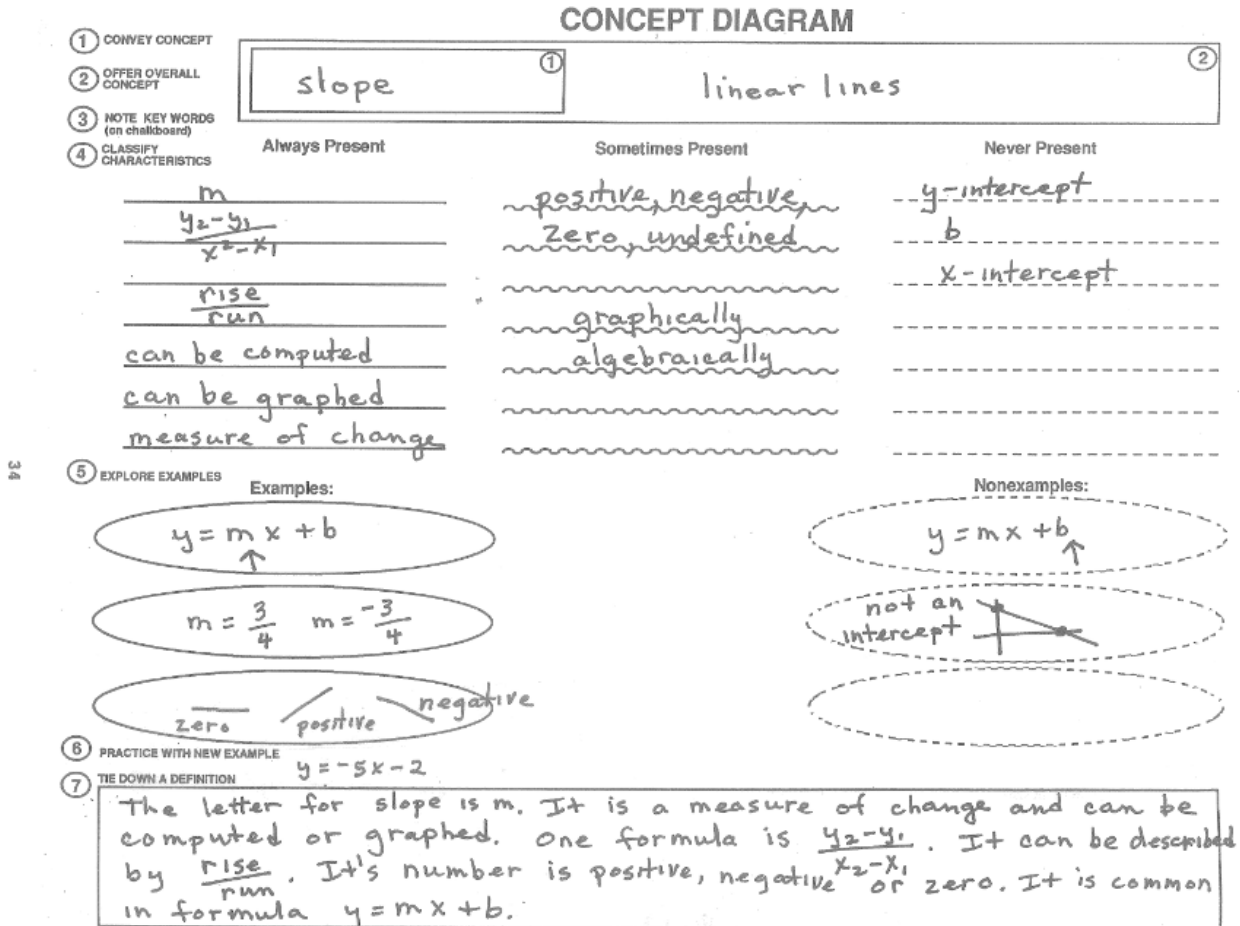
The domain is the set of all inputs in ordered pairs. The domain is the set of x-values and is associated with the x-axis. The domain is described using the independent variable. The domain is the first member of an ordered pair.

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## Appendix A: Concept Routines

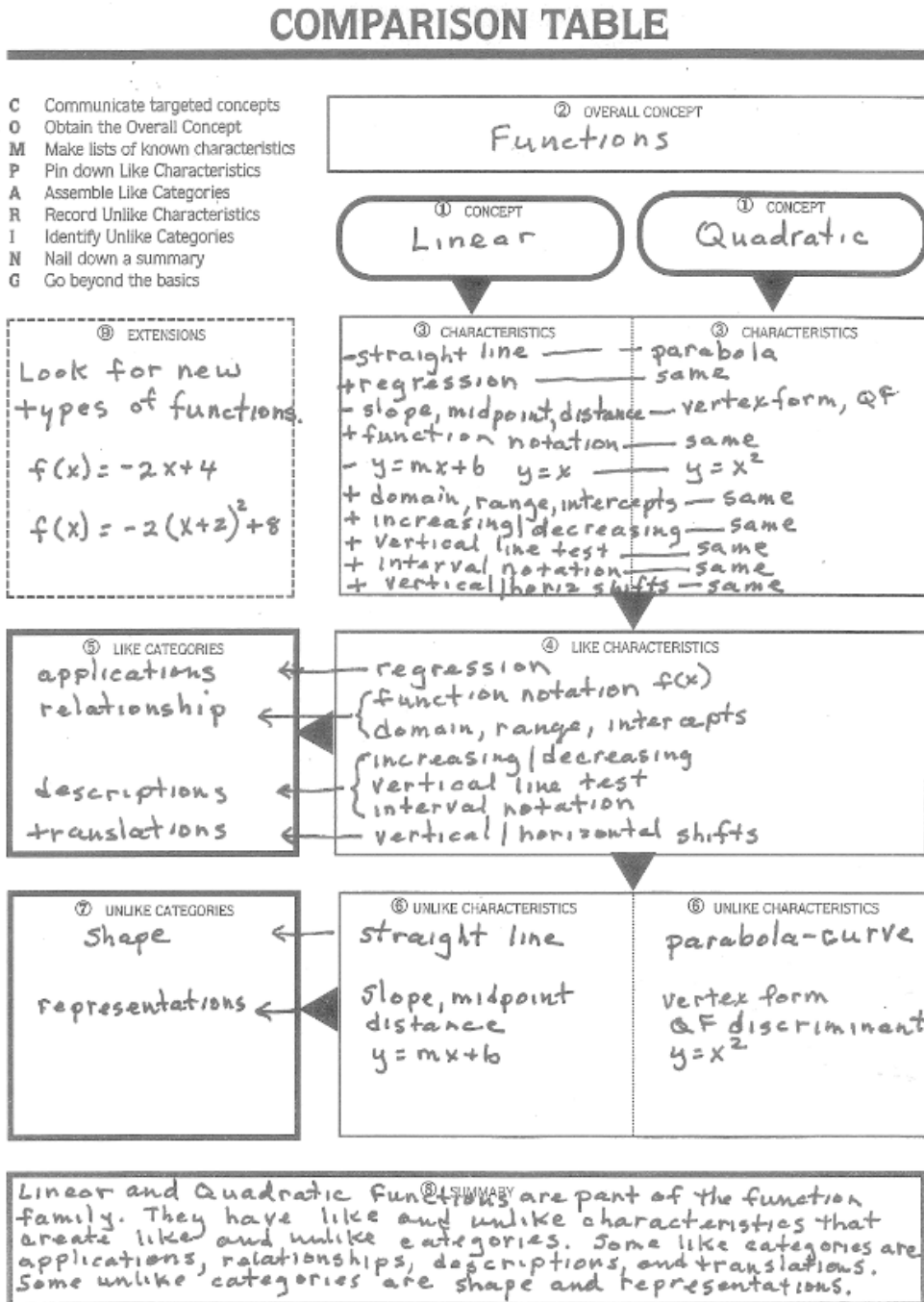
### Concept Diagram: Slope



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## Appendix A: Concept Routines

### Comparison Table



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## Appendix B: Course Organizer Routines

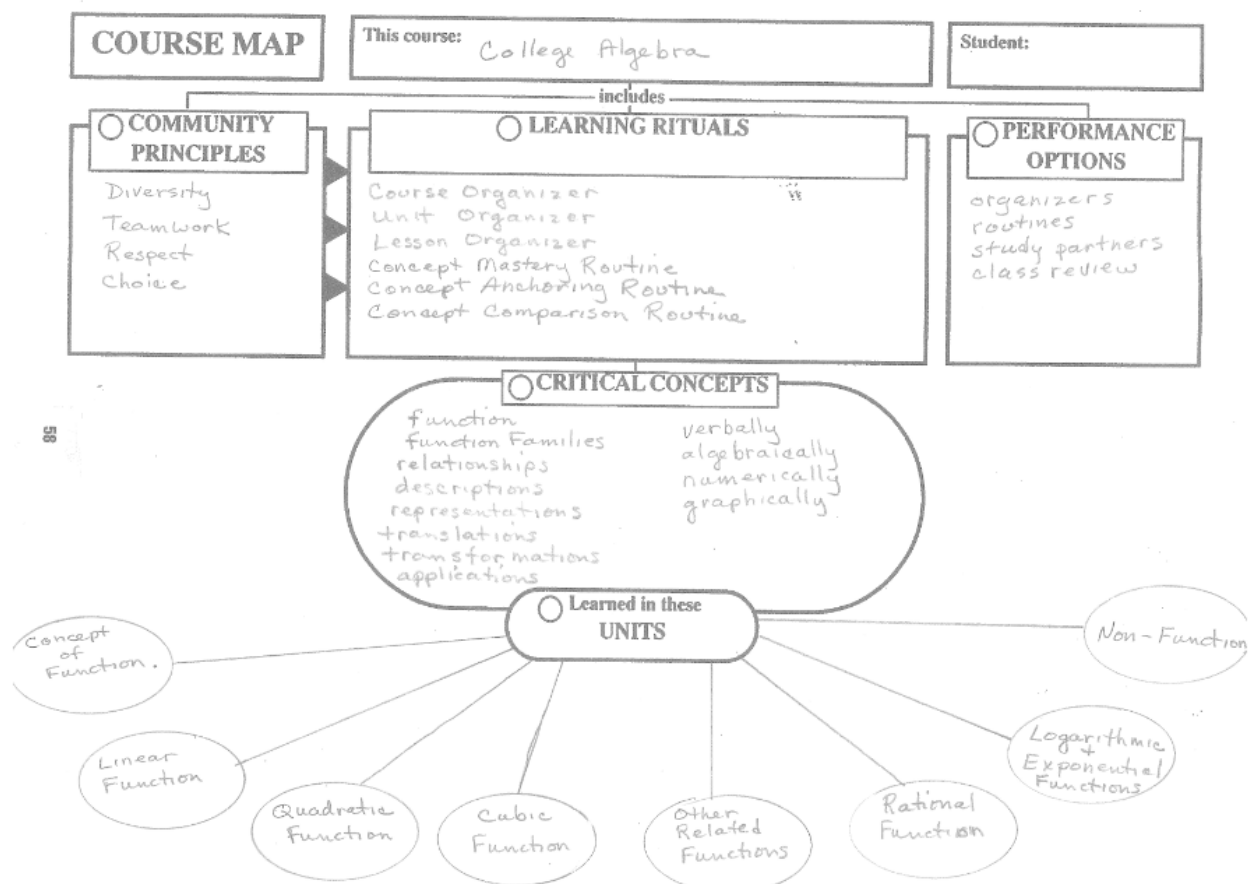
### Course Organizer

The Course Organizer																																		
<div style="border: 1px solid black; padding: 2px; margin-bottom: 5px;">Teacher(s):</div> <div style="border: 1px solid black; padding: 2px;">Time:</div>	<div style="border: 1px solid black; padding: 2px; margin-bottom: 5px;">Student:</div> <div style="border: 1px solid black; padding: 2px;">Course Dates:</div>																																	
<p><input type="radio"/> THIS COURSE: College Algebra</p> <div style="border: 1px solid black; border-radius: 15px; padding: 10px; margin: 10px 0;"> <div style="display: inline-block; border: 1px solid black; border-radius: 50%; padding: 2px 5px; text-align: center;">is about</div> <p>how algebra is divided into different types of function families and how their differences/similarities help solve real life applications.</p> </div>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <input type="radio"/> COURSE STANDARDS:         </div> <table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left; width: 30%;">What?</th> <th style="text-align: left; width: 30%;">How?</th> <th style="text-align: left; width: 40%;">Value?</th> </tr> </thead> <tbody> <tr> <td colspan="3"><b>CONTENT:</b></td> </tr> <tr> <td>Concepts</td> <td>tests</td> <td>75%</td> </tr> <tr> <td></td> <td>homework</td> <td>10%</td> </tr> <tr> <td></td> <td>routines</td> <td>15%</td> </tr> <tr> <td colspan="3"><b>PROCESS:</b></td> </tr> <tr> <td colspan="3">participation - use organizers</td> </tr> <tr> <td colspan="3">- use concept mastery</td> </tr> <tr> <td colspan="3">anchoring</td> </tr> <tr> <td colspan="3">comparison</td> </tr> <tr> <td colspan="3">routines</td> </tr> </tbody> </table>	What?	How?	Value?	<b>CONTENT:</b>			Concepts	tests	75%		homework	10%		routines	15%	<b>PROCESS:</b>			participation - use organizers			- use concept mastery			anchoring			comparison			routines		
What?	How?	Value?																																
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<p><input type="radio"/> COURSE QUESTIONS:</p> <ol style="list-style-type: none"> <li>1. How are relationships identified as functions or non-functions?</li> <li>2. What characteristics are used to describe functions and non-functions?</li> <li>3. What are the main types of functions?</li> <li>4. What are the ways functions and non-functions can be represented mathematically? State the advantage of each representation.</li> <li>5. How are graphs and equations of functions and non-functions related by shifting and reflecting?</li> <li>6. How are graphs and equations of functions and non-functions related by stretching and shrinking or by changing slope?</li> <li>7. How are applications of functions and non-functions understood using different representations?</li> </ol>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <b>COURSE PROGRESS GRAPH</b> </div> <div style="border: 1px solid black; height: 150px; width: 100%;"></div>																																	

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## Appendix B: Course Organizer Routines

### Course Organizer Map



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## Appendix C: Unit & Lesson Organizer Routines

### Function Unit Organizer

NAME \_\_\_\_\_  
DATE \_\_\_\_\_

④ BIGGER PICTURE

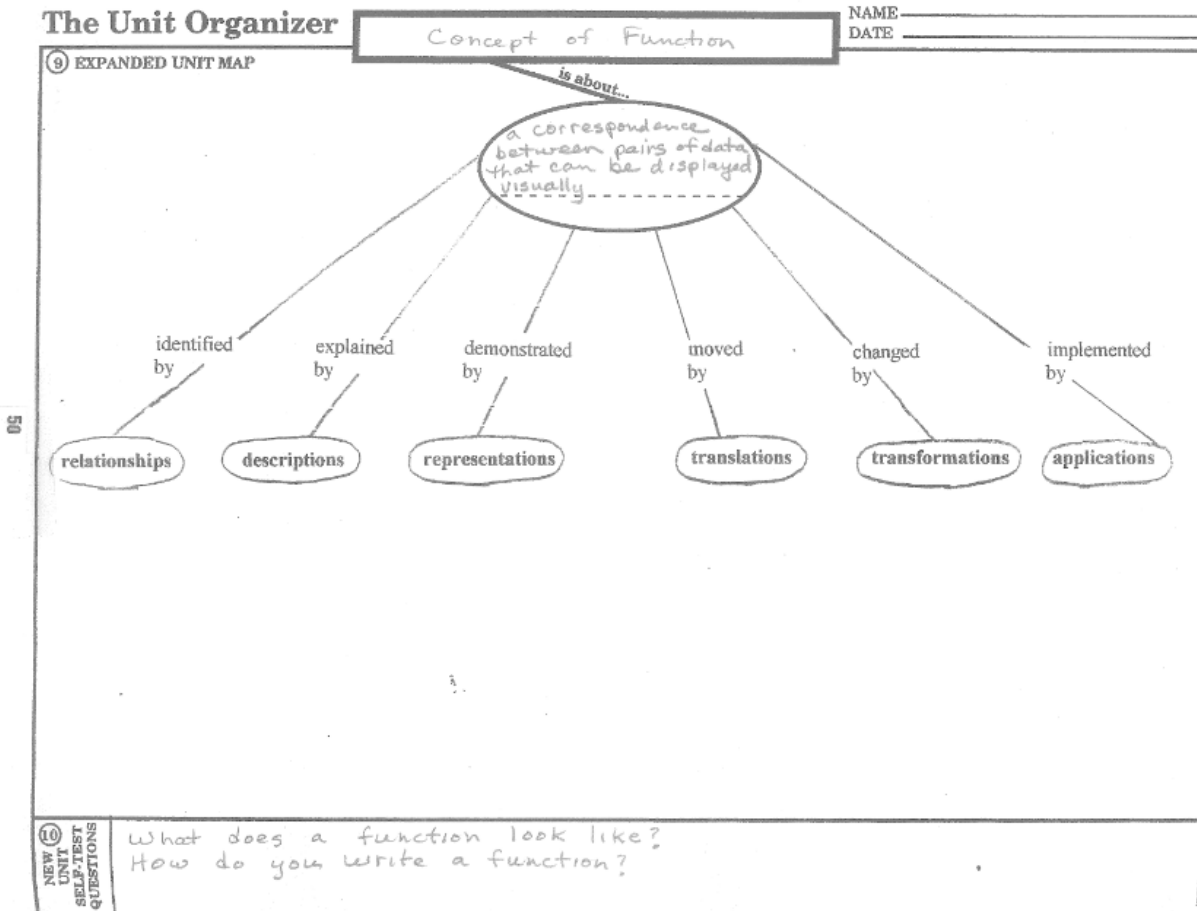
### The Unit Organizer

② LAST UNIT/Experience 	① CURRENT UNIT Concept of Function	③ NEXT UNIT/Experience Linear Function																														
⑧ UNIT SCHEDULE <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>1-2</td><td>relationships</td></tr> <tr><td>1</td><td>descriptions</td></tr> <tr><td></td><td>representations</td></tr> <tr><td>1</td><td>translations</td></tr> <tr><td></td><td>transformations</td></tr> <tr><td>1</td><td>applications</td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </table>	1-2	relationships	1	descriptions		representations	1	translations		transformations	1	applications																			⑤ UNIT MAP <div style="text-align: center;"> <p>is about...</p> </div>	
1-2	relationships																															
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⑦ UNIT SELF-TEST QUESTIONS <ol style="list-style-type: none"> <li>How is a relationship identified as a function?</li> <li>What characteristics are used to describe a function?</li> <li>What are the ways a function can be represented mathematically?</li> <li>How are graphs and equations of functions related by shifting and reflecting?</li> <li>How are graphs and equations of functions related by stretching and shrinking?</li> <li>How are applications understood using different representations?</li> </ol>	⑥ UNIT RELATIONSHIPS <table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td>steps</td></tr> <tr><td>compare/contrast</td></tr> <tr><td>descriptions</td></tr> <tr><td>analyze</td></tr> </table>		steps	compare/contrast	descriptions	analyze																										
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## Appendix C: Unit & Lesson Organizer Routines

### Function Unit Organizer Map



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## Appendix C: Unit & Lesson Organizer Routines

### Function Lesson Organizer: Relationships

Lesson Organizer		④ UNIT or BACKGROUND	DATE: _____	NAME: _____
<div style="text-align: center;"> <b>CONCEPT OF FUNCTION</b> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div>identified by <b>relationships</b></div> <div>explained by <b>descriptions</b></div> <div>demonstrated by <b>representations</b></div> <div>moved by <b>translations</b></div> <div>changed by <b>transformations</b></div> <div>implemented by <b>applications</b></div> </div>				
② Relationships	① LESSON TOPIC		③ Task-Related Strategies	
compare	Relationships			
⑤ Lesson Map <div style="text-align: center; margin-top: 20px;"> <pre> graph TD     A([a correspondence between pairs of data called domain and range])     B([function notation]) -- named by --&gt; A     C([input/output]) -- defined by --&gt; A     D([domain/range]) -- defined by --&gt; A     E([algebraic manipulation]) -- evaluated by --&gt; A     F([graphing calculator]) -- evaluated by --&gt; A     G([graphing calculator]) -- displayed by --&gt; A     H([vertical line test]) -- determined by --&gt; A           </pre> </div>				
⑥ Challenge Question What will the calculator show?				
⑦ Self-Test Questions		⑧ Tasks:		
1. How do you determine if a correspondence (non-ordered pairs) is a function. 2. How do you determine if a correspondence of ordered pairs is a function. 3. How do you determine the domain and range of ordered pairs. 4. How do you determine if a graph is a function using the vertical line test. 5. How do you name a function by determining the dependent and independent variables. 6. How do you evaluate a function.		Homework problems in book calculator		

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## Appendix C: Unit & Lesson Organizer Routines

### Function Lesson Organizer: Descriptions

**Lesson Organizer**

④ UNIT or BACKGROUND: \_\_\_\_\_ DATE: \_\_\_\_\_ NAME: \_\_\_\_\_

**CONCEPT OF FUNCTION**

identified by: relationships  
explained by: descriptions  
demonstrated by: representations  
moved by: translations  
changed by: transformations  
implemented by: applications

② Relationships	① LESSON TOPIC	③ Task-Related Strategies
analyze	Descriptions	

⑤ Lesson Map

is about

recording the analysis of a function by looking at ordered pairs / graphs / equation

using: interval notation

to find: domain / range

to find: discrete / continuous

to find: increasing / decreasing

to find: intercepts

to find: symmetry

⑥ Challenge Question: What will the calculator show?

⑦ Self-Test Questions

1. How do you determine if a graph is symmetric to the x-axis, y-axis, and origin by folding over the axes.
2. How do you determine if an equation is symmetric with respect to the x-axis, y-axis, and origin.
3. How do you determine if a given function is even, odd, or neither.
4. How do you determine the x- and y-intercepts.
5. How do you record interval notation.

⑧ Tasks:

Use index of book to find page numbers of topics.  
calculator  
homework

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## Appendix C: Unit & Lesson Organizer Routines

### Function Lesson Organizer: Representations

Lesson Organizer		④ UNIT or BACKGROUND CONCEPT OF FUNCTION	DATE: _____ NAME: _____
identified by <b>relationships</b>	explained by <b>descriptions</b>	demonstrated by <b>representations</b>	moved by <b>translations</b>
			changed by <b>transformations</b>
			implemented by <b>applications</b>
② Relationships <i>compare</i>	① LESSON TOPIC <b>Representations</b>		③ Task-Related Strategies
<div style="display: flex; justify-content: space-between;"> <div style="width: 30%;"> <p>⑤ Lesson Map</p> <pre> graph TD     A([explaining or exhibiting functions in different ways]) -- is about --- B[ ]     A -- such as --- C([verbally])     A -- such as --- D([algebraically])     A -- such as --- E([numerically])     A -- such as --- F([graphically])     C -- by using --- G([finite steps])     D -- by using --- H([equations])     E -- by using --- I([charts &amp; tables])     F -- by using --- J([graphing calculator])                     </pre> </div> <div style="width: 65%; border-top: 1px dashed black; padding-top: 10px;"> <p>⑥ Challenge Question <i>Do these say the same thing?</i></p> <p>⑦ Self-Test Questions</p> <ol style="list-style-type: none"> <li>1. How do you interpret functions verbally.</li> <li>2. How do you interpret functions algebraically.</li> <li>3. How do you interpret functions numerically.</li> <li>4. How do you interpret functions graphically.</li> </ol> </div> <div style="width: 30%; padding-top: 10px;"> <p>⑧ Tasks:</p> <p><i>calculator</i> <i>use index</i> <i>Review algebra rules</i> <i>homework</i></p> </div> </div>			

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## Appendix C: Unit & Lesson Organizer Routines

### Function Lesson Organizer: Translations

**Lesson Organizer**

④ UNIT or BACKGROUND: **CONCEPT OF FUNCTION** DATE: \_\_\_\_\_ NAME: \_\_\_\_\_

identified by: **relationships** explained by: **descriptions** demonstrated by: **representations** moved by: **translations** changed by: **transformations** implemented by: **applications**

② Relationships	① LESSON TOPIC	③ Task-Related Strategies
<i>compare/contrast</i>	<b>Translations</b>	

⑤ Lesson Map

*is about*

*shifting the position of functions by altering the equation/graph*

*such as*

*reflecting* *horizontal* *vertical*

⑥ Challenge Question *Will the calculator give the equation?*

⑦ Self-Test Questions

1. How do you relate graphs of functions by horizontal and vertical shifting.
2. How do you relate graphs of functions by reflecting.
3. How do you write equations of functions given shifting and reflecting.
4. How do you graph translations of functions given the basic graph, shifting, and reflecting.

⑧ Tasks:

*calculator record graphs homework*

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## Appendix C: Unit & Lesson Organizer Routines

### Function Lesson Organizer: Transformations

**Lesson Organizer**

④ UNIT or BACKGROUND **CONCEPT OF FUNCTION** DATE: \_\_\_\_\_ NAME: \_\_\_\_\_

identified by relationships  
explained by descriptions  
demonstrated by representations  
moved by translations  
changed by transformations  
implemented by applications

② Relationships	① LESSON TOPIC	③ Task-Related Strategies
compare	Transformations	

⑤ Lesson Map

Is about

changing the shape of functions by altering the equation/graph

by stretching

by shrinking

⑥ Challenge Question How does the calculator help?

⑦ Self-Test Questions

1. How do you relate graphs of functions by stretching and shrinking.
2. How do you write equations of functions given stretching and shrinking.
3. How do you graph transformations of functions given the basic graph, stretching, and shrinking.

⑧ Tasks:

calculator  
record graphs  
homework

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## Appendix C: Unit & Lesson Organizer Routines

### Function Lesson Organizer: Applications

Lesson Organizer		④ UNIT or BACKGROUND CONCEPT OF FUNCTION	DATE: _____ NAME: _____
identified by relationships	explained by descriptions	demonstrated by representations	moved by translations
			changed by transformations
			implemented by applications
② Relationships	① LESSON TOPIC	③ Task-Related Strategies	
compare/contrast	Applications		
⑤ Lesson Map			
<p>Is about</p> <pre> graph TD     A([seeing the concept of function in real-life problems or data]) -- by --&gt; B([numerically])     A -- by --&gt; C([graphically])     A -- by --&gt; D([algebraically])     B -- using --&gt; E([tables])     C -- using --&gt; F([scatterplots])     D -- using --&gt; G([regression equation])         </pre>			
⑥ Challenge Question Explain how these are connected.			
⑦ Self-Test Questions		⑧ Tasks:	
<ol style="list-style-type: none"> <li>How do you create a scatterplot using given data.</li> <li>How do you determine the type of curve of a given scatterplot.</li> <li>How do you determine an equation for given data.</li> <li>How do you use the regression feature on the calculator to find missing data.</li> </ol>		homework calculator	

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## Appendix C: Unit & Lesson Organizer Routines

### Linear Unit Organizer

**The Unit Organizer** NAME \_\_\_\_\_  
DATE \_\_\_\_\_

④ BIGGER PICTURE

② LAST UNIT/Experience Concept of Function	① CURRENT UNIT Linear Function	③ NEXT UNIT/Experience Quadratic Function
---	-----------------------------------	--

⑧ UNIT SCHEDULE

1 relationships
1 descriptions
1 representations
1 translations
1 transformations
1 applications

⑤ UNIT MAP

⑦ UNIT SELF-TEST QUESTIONS

1. How is a relationship identified as a linear function?
2. What characteristics are used to describe a linear function?
3. What are the ways a linear function can be represented mathematically?
4. How are graphs and equations of linear functions related by shifting and reflecting?
5. How are graphs and equations of linear functions related by changing slope?
6. How are linear applications understood using different representations?

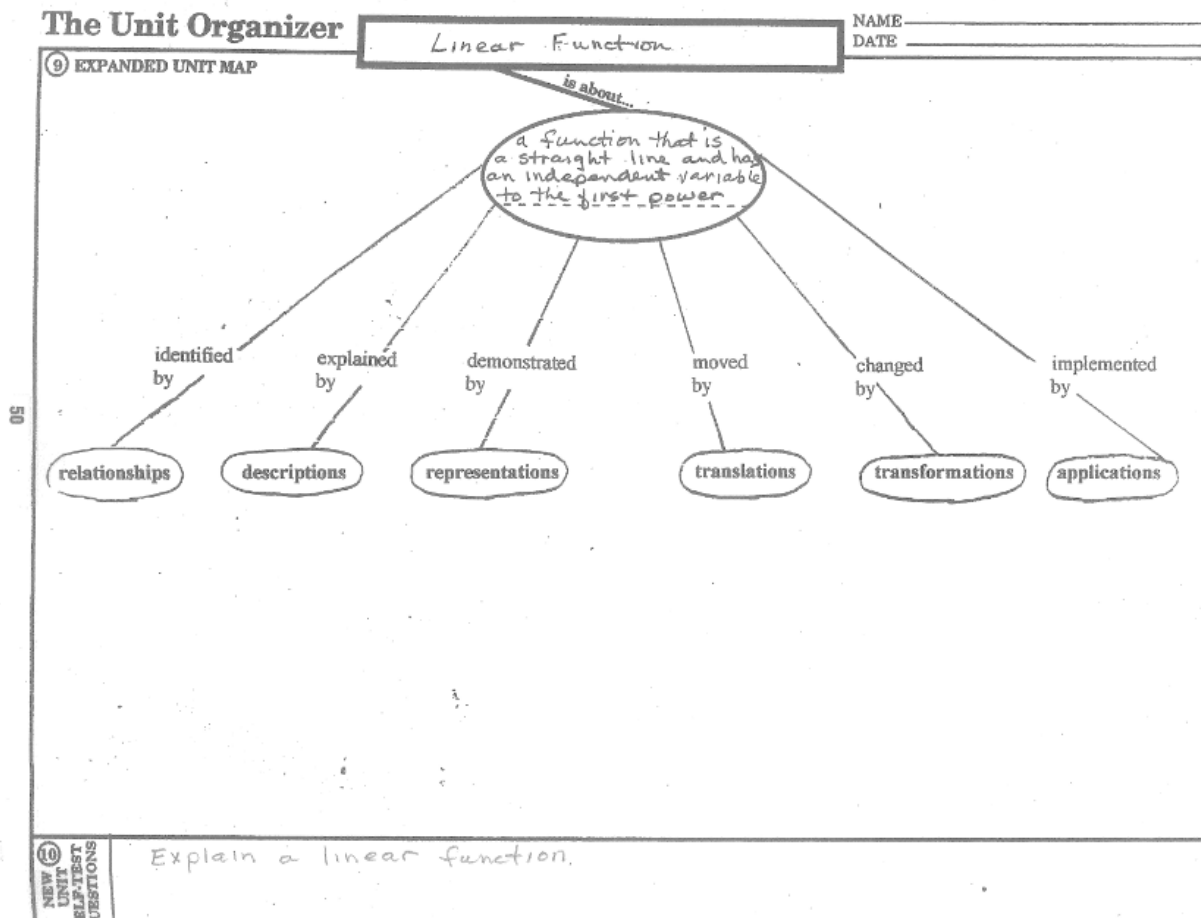
⑥ UNIT RELATIONSHIPS

problems
comparisons
procedures

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## Appendix C: Unit & Lesson Organizer Routines

### Linear Unit Organizer Map



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## Appendix C: Unit & Lesson Organizer Routines

### Linear Lesson Organizer: Relationships

**Lesson Organizer**

④ UNIT or BACKGROUND: **LINEAR FUNCTION** DATE: \_\_\_\_\_ NAME: \_\_\_\_\_

identified by: **relationships** explained by: **descriptions** demonstrated by: **representations** moved by: **translations** changed by: **transformations** implemented by: **applications**

② Relationships	① LESSON TOPIC	③ Task-Related Strategies
compare	Relationships	

⑤ Lesson Map

is about: a correspondence between pairs of data that form a linear line

named by: function notation

defined by: input/output

defined by: domain/range

evaluated by: algebraic manipulation

evaluated by: graphing calculator

displayed by: graphing calculator

determined by: vertical line test

⑥ Challenge Question: What else can the calculator do?

⑦ Self-Test Questions

1. How do you determine if a graph/equation is a linear function.
2. How do you determine the domain and range of a linear function.
3. How do you evaluate a linear function using function notation.
4. How do you display the graph of a linear function.
5. How do you read the graph of a linear function.

⑧ Tasks: Homework calculator

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## Appendix C: Unit & Lesson Organizer Routines

### Linear Lesson Organizer: Descriptions

Lesson Organizer		④ UNIT or BACKGROUND	DATE: _____	NAME: _____
<div style="text-align: center; border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <b>LINEAR FUNCTION</b> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;">identified by <b>relationships</b></div> <div style="text-align: center;">explained by <b>descriptions</b></div> <div style="text-align: center;">demonstrated by <b>representations</b></div> <div style="text-align: center;">moved by <b>translations</b></div> <div style="text-align: center;">changed by <b>transformations</b></div> <div style="text-align: center;">implemented by <b>applications</b></div> </div>				
② Relationships	① LESSON TOPIC		③ Task-Related Strategies	
<i>analyze</i>	Descriptions			
<div style="border: 1px solid black; padding: 10px; margin-top: 10px;"> <b>⑤ Lesson Map</b>  <div style="text-align: center; margin-top: 20px;"> <p><i>Is about</i></p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; display: inline-block; text-align: center;">             recording the analysis of a linear function by looking at ordered pairs graphs/equations           </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <p><i>using</i></p> <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">interval notation</div> </div> <div style="text-align: center;"> <p><i>to find</i></p> <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">domain/ range</div> </div> <div style="text-align: center;"> <p><i>to find</i></p> <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">discrete/ continuous</div> </div> <div style="text-align: center;"> <p><i>to find</i></p> <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">increasing/ decreasing</div> </div> <div style="text-align: center;"> <p><i>to find</i></p> <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">intercepts</div> </div> <div style="text-align: center;"> <p><i>to find</i></p> <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">symmetry</div> </div> </div> </div>				
<b>⑥ Challenge Question</b> <i>How do you alter graphs on the calculator?</i>				
<b>⑦ Self-Test Questions</b> <ol style="list-style-type: none"> <li>How do you determine domain/range, increasing/decreasing, and continuous/discrete of a linear function and record using interval notation.</li> <li>How do you determine the x and y-intercepts of a linear functions.</li> <li>How do you determine if a linear function is even, odd, or neither.</li> <li>How do you determine the symmetry of a linear function</li> </ol>		<b>⑧ Tasks:</b> <i>Use index of book. Homework problems calculator</i>		

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## Appendix C: Unit & Lesson Organizer Routines

### Linear Lesson Organizer: Representations

UNIT or BACKGROUND		DATE: _____	NAME: _____
<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-bottom: 10px;"><b>LINEAR FUNCTION</b></div> <div style="display: flex; justify-content: space-around; font-size: small;"> <span>identified by</span> <span>explained by</span> <span>demonstrated by</span> <span>moved by</span> <span>changed by</span> <span>implemented by</span> </div> <div style="display: flex; justify-content: space-around; text-align: center;"> <div style="border: 1px solid black; border-radius: 15px; padding: 2px 10px;">relationships</div> <div style="border: 1px solid black; border-radius: 15px; padding: 2px 10px;">descriptions</div> <div style="border: 1px solid black; border-radius: 15px; padding: 2px 10px;">representations</div> <div style="border: 1px solid black; border-radius: 15px; padding: 2px 10px;">translations</div> <div style="border: 1px solid black; border-radius: 15px; padding: 2px 10px;">transformations</div> <div style="border: 1px solid black; border-radius: 15px; padding: 2px 10px;">applications</div> </div>			
<b>② Relationships</b> <i>compare</i>	<b>① LESSON TOPIC</b> <b>Representations</b>	<b>③ Task-Related Strategies</b>	
<b>⑤ Lesson Map</b> <div style="text-align: center; margin-top: 20px;"> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; display: inline-block;"> <i>explaining or exhibiting linear functions in different ways</i> </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;"> <i>equations</i>  <i>using</i> </div> <div style="text-align: center;"> <i>slopes</i>  <i>using</i> </div> <div style="text-align: center;"> <i>intercepts</i>  <i>using</i> </div> <div style="text-align: center;"> <i>midpoint</i>  <i>using</i> </div> <div style="text-align: center;"> <i>distance</i>  <i>using</i> </div> <div style="text-align: center;"> <i>inequalities</i>  <i>using</i> </div> </div>			
<b>⑥ Challenge Question</b> <i>How do these differ?</i>			
<b>⑦ Self-Test Questions</b> <ol style="list-style-type: none"> <li>1. How do you determine the slope of a line.</li> <li>2. How do you determine the slope and y-intercept of a line given a linear equation.</li> <li>3. How do you write a slope-intercept equation given slope and y-intercept or two points.</li> <li>4. How do you determine if lines are parallel, perpendicular, or neither.</li> <li>5. How do you determine the distance between two points using the distance formula.</li> <li>6. How do you determine the midpoint of a line segment joining two points.</li> <li>7. How do you solve an equation/inequality of a linear function.</li> </ol>		<b>⑧ Tasks:</b> <i>calculator</i> <i>use index</i> <i>algebra rules</i> <i>homework</i>	

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## Appendix C: Unit & Lesson Organizer Routines

### Linear Lesson Organizer: Translations

④ UNIT or BACKGROUND		DATE: _____	NAME: _____
<div style="text-align: center; border: 1px solid black; padding: 5px; margin: 10px auto; width: 200px;"> <b>LINEAR FUNCTION</b> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;">identified by <b>relationships</b></div> <div style="text-align: center;">explained by <b>descriptions</b></div> <div style="text-align: center;">demonstrated by <b>representations</b></div> <div style="text-align: center;">moved by <b>translations</b></div> <div style="text-align: center;">changed by <b>transformations</b></div> <div style="text-align: center;">implemented by <b>applications</b></div> </div>			
② Relationships	① LESSON TOPIC		③ Task-Related Strategies
compare/contrast	Translations		
⑤ Lesson Map <div style="text-align: center; margin-top: 20px;"> <p>is about</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; display: inline-block; text-align: center;">             shifting the position of linear functions by altering the equation/ graph           </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;">such as <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">reflecting</div></div> <div style="text-align: center;">such as <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">horizontal</div></div> <div style="text-align: center;">such as <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">vertical</div></div> </div> </div>			
⑥ Challenge Question How do you change windows?			
⑦ Self-Test Questions		⑧ Tasks:	
1. How do you relate graphs of linear functions by horizontal and vertical shifting. 2. How do you relate graphs of linear functions by reflecting. 3. How do you write equations of linear functions given shifting and reflecting. 4. How do you graph translations of linear functions given the basic graph, shifting, and reflecting.		calculator record graphs homework	

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## Appendix C: Unit & Lesson Organizer Routines

### Linear Lesson Organizer: Transformations

④ UNIT or BACKGROUND			DATE: _____	NAME: _____
identified by relationships	explained by descriptions	demonstrated by representations	moved by translations	changed by transformations
implemented by applications				
② Relationships	① LESSON TOPIC	③ Task-Related Strategies		
compare	Transformations			
⑤ Lesson Map				
<p>changing the shape of linear functions by altering the equation</p> <p>by positive slope      by negative slope      by zero slope</p>				
⑥ Challenge Question: Will the calculator give you the equation?				
⑦ Self-Test Questions		⑧ Tasks:		
1. How do you relate graphs of linear functions by changing slope? 2. How do you write equations of linear functions given different slopes? 3. How do you graph transformations of linear functions given the basic graph and different slopes?		calculator record graphs homework		

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## Appendix C: Unit & Lesson Organizer Routines

### Linear Lesson Organizer: Applications

⑤ UNIT or BACKGROUND		DATE: _____	NAME: _____
<div style="text-align: center;"> <b>LINEAR FUNCTION</b> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: center;">identified by <b>relationships</b></div> <div style="text-align: center;">explained by <b>descriptions</b></div> <div style="text-align: center;">demonstrated by <b>representations</b></div> <div style="text-align: center;">moved by <b>translations</b></div> <div style="text-align: center;">changed by <b>transformations</b></div> <div style="text-align: center;">implemented by <b>applications</b></div> </div>			
② Relationships	① LESSON TOPIC	③ Task-Related Strategies	
compare/contrast	Applications		
⑥ Lesson Map <div style="text-align: center; margin-top: 20px;"> <pre> graph TD     A([Seeing the concept of linear functions in real-life problems or data.]) -- "by" --&gt; B([numerically])     A -- "by" --&gt; C([graphically])     A -- "by" --&gt; D([algebraically])     B -- "using" --&gt; E([tables])     C -- "using" --&gt; F([scatterplots])     D -- "using" --&gt; G([regression equation])           </pre> </div>			
⑥ Challenge Question: Explain how these are connected.			
⑦ Self-Test Questions		⑧ Tasks:	
1. How do you write an equation for a linear application. 2. How do you determine the domain for a linear application. 3. How do you solve an equation given a linear application. 4. How do you determine if a scatterplot represents a linear function. 5. How do you find the midpoint of a line segment in a linear application. 6. How do you solve an inequality given a linear application.		homework calculator	

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## Appendix C: Unit & Lesson Organizer Routines

### Quadratic Unit Organizer

**The Unit Organizer** NAME \_\_\_\_\_  
DATE \_\_\_\_\_

④ BIGGER PICTURE

② LAST UNIT/Experience Linear Function	① CURRENT UNIT <b>Quadratic Function</b>	③ NEXT UNIT/Experience Cubic Function
---	---	--

⑧ UNIT SCHEDULE

1 relationships
1 descriptions
1 representations
1 translations
1 transformations
1 applications

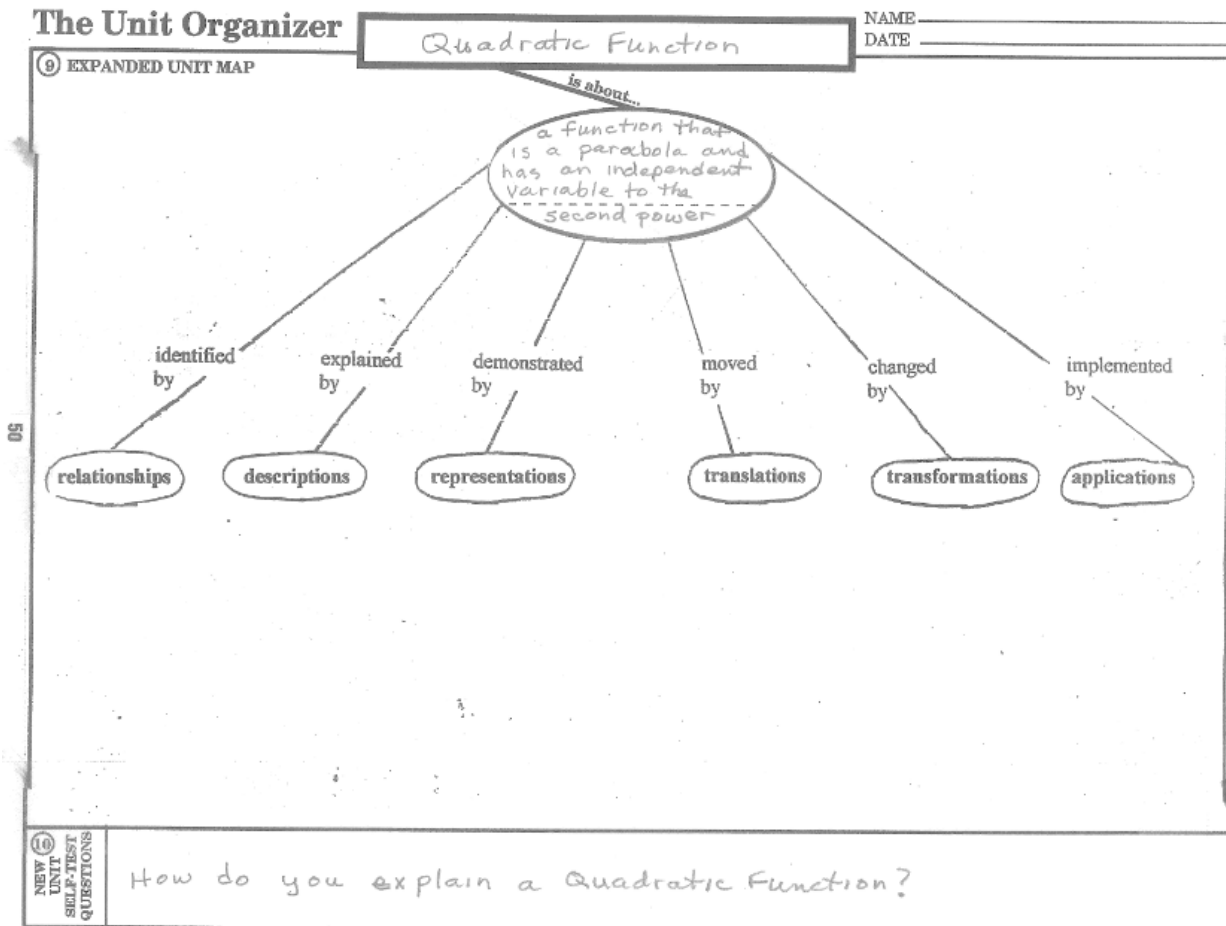
⑤ UNIT MAP

⑦ UNIT SELF-TEST QUESTIONS	<ol style="list-style-type: none"> <li>1. How is a relationship identified as a quadratic function?</li> <li>2. What characteristics are used to describe a function?</li> <li>3. What are the ways a quadratic function can be represented mathematically?</li> <li>4. How are graphs and equations of a quadratic function related by shifting and reflecting?</li> <li>5. How are graphs and equations of quadratic functions related by stretching and shrinking?</li> <li>6. How are quadratic applications understood using different representations?</li> </ol>	⑥ UNIT RELATIONSHIPS problems comparisons procedures .
----------------------------	---	--

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## Appendix C: Unit & Lesson Organizer Routines

### Quadratic Unit Organizer Map



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## Appendix C: Unit & Lesson Organizer Routines

### Quadratic Lesson Organizer: Relationships

Lesson Organizer		④ UNIT or BACKGROUND QUADRATIC FUNCTION	DATE: _____ NAME: _____
identified by <b>relationships</b>	explained by <b>descriptions</b>	demonstrated by <b>representations</b>	moved by <b>translations</b>
changed by <b>transformations</b>	implemented by <b>applications</b>		
② Relationships <i>analyze</i>	① LESSON TOPIC <b>Relationships</b>	③ Task-Related Strategies	
⑤ Lesson Map			
<pre> graph TD     A([a correspondence between pairs of data that form a parabola])     B([function notation]) -- named by --&gt; A     C([input/output]) -- defined by --&gt; A     D([domain/range]) -- defined by --&gt; A     E([algebraic manipulation]) -- evaluated by --&gt; A     F([graphing calculator]) -- evaluated by --&gt; A     G([vertical line]) -- determined by --&gt; A     H([graphing calculator]) -- displayed by --&gt; A     </pre>			
⑥ Challenge Question <i>How do we adjust windows?</i>			
⑦ Self-Test Questions		⑧ Tasks:	
<ol style="list-style-type: none"> <li>How do you determine if a graph/equation is a quadratic function.</li> <li>How do you determine the domain and range of a quadratic function.</li> <li>How do you evaluate a quadratic function given function notation.</li> <li>How do you display the graph of a quadratic function.</li> <li>How do you read a graph of a quadratic function.</li> </ol>		<i>Homework Calculator</i>	

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## Appendix C: Unit & Lesson Organizer Routines

### Quadratic Lesson Organizer: Descriptions

**Lesson Organizer**

**④ UNIT or BACKGROUND** **QUADRATIC FUNCTION** DATE: \_\_\_\_\_ NAME: \_\_\_\_\_

identified by: **relationships** explained by: **descriptions** demonstrated by: **representations** moved by: **translations** changed by: **transformations** implemented by: **applications**

**② Relationships** **① LESSON TOPIC** **③ Task-Related Strategies**

*analyze* **Descriptions**

**⑥ Lesson Map**

*Is about:*

interval notation *using* **recording the analysis of a quadratic function by looking at ordered pairs, graphs/equations** *to find* symmetry

*to find* domain/range *to find* discrete/continuous *to find* increasing/decreasing *to find* intercepts

**⑥ Challenge Question** *How do you zoom in or out on the calculator?*

**⑦ Self-Test Questions** **⑧ Tasks:**

1. How do you determine domain/range, increasing/decreasing, and continuous/discrete of a quadratic function and record using interval notation.
2. How do you determine relative maximum or minimum of a quadratic function.
3. How do you determine the zeroes of a quadratic function.
4. How do you determine if a quadratic function is even, odd, or neither.
5. How do you determine the symmetry of a quadratic function.

*Index of book  
Homework  
calculator*

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## Appendix C: Unit & Lesson Organizer Routines

### Quadratic Lesson Organizer: Representations

**Lesson Organizer**

④ UNIT or BACKGROUND: **QUADRATIC FUNCTION** DATE: \_\_\_\_\_ NAME: \_\_\_\_\_

identified by: **relationships** explained by: **descriptions** demonstrated by: **representations** moved by: **translations** changed by: **transformations** implemented by: **applications**

② Relationships: **compare/contrast** ① LESSON TOPIC: **Representations** ③ Task-Related Strategies: \_\_\_\_\_

⑥ Lesson Map

is about: **explaining or exhibiting quadratic functions in different ways**

using: **equations** **discriminant** **vertex** **symmetry** **intercepts** **inequalities**

⑦ Challenge Question: **How do you use trace on the calculator?**

⑦ Self-Test Questions: ⑧ Tasks:

- How do you determine the number of zeroes of a quadratic function.
- How do you use the discriminant to determine the types of solutions.
- How do you determine the vertex of a parabola.
- How do you determine the line of symmetry of a parabola.
- How do you solve an equation of a quadratic function for an exact answer.
- How do you solve an equation of a quadratic function for an approximation.
- How do you solve an inequality of a quadratic function.

calculator  
Algebra rules & formulas  
Homework

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## Appendix C: Unit & Lesson Organizer Routines

### Quadratic Lesson Organizer: Translations

Lesson Organizer		④ UNIT or BACKGROUND QUADRATIC FUNCTION	DATE: _____ NAME: _____
identified by <b>relationships</b>	explained by <b>descriptions</b>	demonstrated by <b>representations</b>	moved by <b>translations</b>
			changed by <b>transformations</b>
			implemented by <b>applications</b>
② Relationships <i>compare/contrast</i>	① LESSON TOPIC <b>Translations</b>	③ Task-Related Strategies	
⑤ Lesson Map			
⑥ Challenge Question <i>How do you put two equations on the same graph?</i>			
⑦ Self-Test Questions		⑧ Tasks:	
<ol style="list-style-type: none"> <li>1. How do you relate graphs of quadratic functions by shifting.</li> <li>2. How do you relate graphs of quadratic functions by reflecting.</li> <li>3. How do you write equation of quadratic functions given shifting and reflecting.</li> <li>4. How do you graph translations of quadratic functions given the basic graph, shifting, and reflecting.</li> </ol>		<i>calculator</i> <i>Record graphs</i> <i>Homework</i>	

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## Appendix C: Unit & Lesson Organizer Routines

### Quadratic Lesson Organizer: Transformations

Lesson Organizer		④ UNIT or BACKGROUND QUADRATIC FUNCTION	DATE: _____ NAME: _____
identified by <b>relationships</b>	explained by <b>descriptions</b>	demonstrated by <b>representations</b>	moved by <b>translations</b>
			changed by <b>transformations</b>
			implemented by <b>applications</b>
② Relationships <i>compare</i>	① LESSON TOPIC <i>Transformations</i>	③ Task-Related Strategies	
⑤ Lesson Map			
<p style="text-align: center;"> <i>is about</i>  </p>			
⑥ Challenge Question <i>How do you translate and transform?</i>			
⑦ Self-Test Questions		⑧ Tasks:	
<ol style="list-style-type: none"> <li>How do you relate graphs of quadratic functions by stretching and shrinking.</li> <li>How do you write equations of quadratic functions given stretching and shrinking.</li> <li>How do you graph transformations of functions given the basic graph, stretching, and shrinking.</li> </ol>		<i>calculator</i> <i>record graphs</i> <i>homework</i>	

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## Appendix C: Unit & Lesson Organizer Routines

### Quadratic Lesson Organizer: Applications

Lesson Organizer		④ UNIT or BACKGROUND	DATE: _____	NAME: _____
<div style="text-align: center;"> <b>QUADRATIC FUNCTION</b> </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div>identified by <b>relationships</b></div> <div>explained by <b>descriptions</b></div> <div>demonstrated by <b>representations</b></div> <div>moved by <b>translations</b></div> <div>changed by <b>transformations</b></div> <div>implemented by <b>applications</b></div> </div>				
② Relationships	① LESSON TOPIC	③ Task-Related Strategies		
compare/contrast	Applications			
⑤ Lesson Map				
<div style="text-align: center;"> <p>is about</p> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; display: inline-block;">             Saving the concept of quadratic functions in real-life problems or data.           </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div style="text-align: center;">             by  <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">numerically</div>              using  <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">tables</div> </div> <div style="text-align: center;">             by  <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">graphically</div>              using  <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">scatterplots</div> </div> <div style="text-align: center;">             by  <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">algebraically</div>              using  <div style="border: 1px solid black; border-radius: 50%; padding: 5px;">regression equation</div> </div> </div>				
⑥ Challenge Question Explain how these are connected.				
⑦ Self-Test Questions		⑧ Tasks:		
1. How do you write an equation for a quadratic application. 2. How do you determine the domain of a quadratic application. 3. How do you solve an equation given a quadratic application. 4. How do you determine if a scatterplot represents a quadratic function. 5. How do you solve an inequality given a quadratic application.		Homework calculator		

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## Appendix D: Instruments

### Algebra Achievement Test

The mathematics achievement test was designed using the test-generator that was a supplement to the textbook. The test consisted of thirty-three multiple choice questions with a possibility of four answers.

The thirty-three questions were chosen to correspond to the concepts of a college algebra course. This test covered linear and quadratic functions.

A summary of the questions:

Linear/Quadratic Differences	4 questions
Domain	5 question
Slope	5 questions
Function Attributes	10 questions
Function Notation	2 questions
Inequalities	2 questions
Equations	2 questions
Regression	3 questions

The questions were a combination of topics covering function notation, regression, translations, symmetry, even/odd functions, zeroes, maximum/minimum, inequalities, distance, midpoint, applications and graph analysis.

A typical problem:

What is the equation is symmetric to:

$$f(x) = -3x^2 - 5$$

a)origin      b)y-axis      c)x-axis      d)x-axis, origin

## **Appendix D: Instruments**

### **Confidence in Learning Mathematics Scale**

The confidence in learning mathematics survey was from the Fennema-Sherman Mathematics Attitude Scale. This survey was a set of twelve statements for students to express their feelings. The survey consisted of six positive statements and six negative statements about their confidence in learning mathematics.

The five choices included:

- 1 Strongly Agree
- 2 Agree
- 3 Undecided
- 4 Disagree
- 5 Strongly Disagree

A typical positive statement: Math has been my best subject.

A typical negative statement: Math seems difficult for me.

## **Appendix D: Instruments**

### **Content Enhancement Perceived Value Survey**

This survey was designed by the researcher to access the perceived value of the different Content Enhancement Routines by each of the students. It was modeled after a survey used at the University of Akron in Ohio (Palagallo, 1999). The students were asked to evaluate how helpful the following routine was in their learning process.

They used the following scale for each of the routines or organizers.

EH     extremely helpful

MH     moderately helpful

SH     slightly helpful

NH     not helpful

A sample of the survey question:

Evaluate how helpful the following routine was in your learning process.

Use the following scale and circle your perceived value of the routine:

EH=extremely helpful  
MH=moderately helpful  
SH=slightly helpful  
NH=not helpful

**The Concept Comparison**

**EH    MH    SH    NH**

**(Linear & Quadratic Functions)**